Warm - up   NO CALCULATOR

1. What is 17.5% of $64.00

2. What are the next three numbers of the following sequences:

   a. 4/15, -1/15, -2/5, -11/15, ... $-\frac{1}{3}$ $-\frac{16}{15}$, $\frac{-7}{5}$, $\frac{-26}{25}$

   b. 1, 1, 2, 3, 5, 8, ... 13, 21, 34

Objective: Solve and identify arithmetic sequence and series.

The sequence is an ordered progression of numbers that follows some type of pattern.

A finite sequence ends
   5, 10, 15, 20, 25, 30

An infinite sequence does not end.
   5, 10, 15, 20, 25, ...

Notation:

a_1 means FIRST TERM

a_n means NUMBER OF TERM WE WANT

d means COMMON DIFFERENCE

TERM

n	1	2	3	4	5	6	7	
a_n	a_1	a_2	a_3	- -	- -	- -	-	a_n
SEQ	7	11	15	19	23	27	31	3

d= FOR FOR FOR

$a_1 \quad a_1+4 \quad a_1 + 2(4)$

Arithmetic seqences add a common different to each term, creating the next term.

$a_1, a_1 + d, a_1 + 2d, \ldots, a_1 + (n - 1)d, \ldots$

$a_n = 6 + 4n - 4$

$a_n = 4n + 2$

Formula: $a_n = a_1 + (n - 1)d$

$a_n = dn + c$

For each arithmetic sequence below
a) find the common difference
b) find the 10th term
c) find an explicit rule for the nth term

1. -6, -2, 2, 6, 10, ...

a) d = 4
b) a_{10} = 30
c) a_n = -6 + (n-1)4

2. 6, 10, 14, 18, ...

a) 4
b) a_{10} = 42
c) a_n = 6 + (n-1)4

7) $a_n = -11 + 7n$

Find a_{34}

$a_{34} = 227$

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8) $a_n = 65 - 100n$

Find a_{39}

$a_{39} = -3835$

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9) $a_n = -7.1 - 2.1n$

Find a_{27}

$a_{27} = -63.8$

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11) $a_1 = 28$, $d = 10$

Explicit: $a_n = 18 + 10n$

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12) $a_1 = -38$, $d = -100$

Explicit: $a_n = 62 - 100n$

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13) $a_1 = -34$, $d = -10$
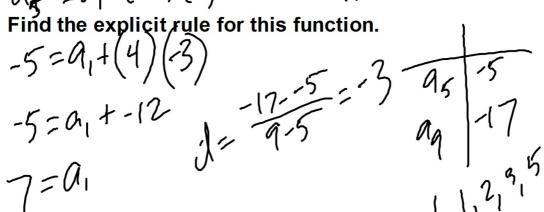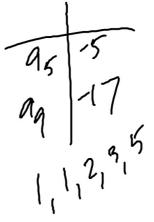
Explicit: $a_n = -24 - 10n$

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30. The fifth and ninth terms of an arithmetic sequence are -5 and -17, respectively. Find the first term and a ~~recursive~~ rule for the nth term.

$$a_n = 7 + (n-1)-3$$

$$a_5 = a_1 + (n-1)(d)$$

Find the explicit rule for this function.

$$-5 = a_1 + (4)(-3)$$

$$-5 = a_1 + -12$$

$$7 = a_1$$

$$d = \frac{-17 - -5}{9 - 5} = -3$$

a_5	-5
a_9	-17

$1, 1, 2, 9, 5$

15) $a_{38} = -53.2$, $d = -1.1$

$a_1 = -12.5$

Explicit: $a_n = -11.4 - 1.1n$

16) $a_{40} = -1191$, $d = -30$

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17) $a_{37} = 249$, $d = 8$

Explicit: $a_n = -47 + 8n$

$a_1 = -39$

$a_n = -39 + (n-1)8$

18) $a_{36} = -276$, $d = -7$

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The first two terms of the arithmetic sequence is given, find the missing term.

1. $a_1 = 5$, $a_2 = 11$, find a_{10}.

2. $a_1 = 4.2$, $a_2 = 6.6$, find a_7.

3. Determine the seating capacity of an auditorium with 30 rows of seats if there are 20 seats in the 1st row, 24 seats in the 2nd row, 28 seats in the 3rd row, and so on.

Add every integer from 1 to 100:
1 + 2 + 3 + 4 + 5 + ... + 100

Sum of an Arithmetic Series

The sum, S_n, of the first n terms of an arithmetic series with first term a_1 and nth term a_n, is given by

$$S_n = \frac{n}{2}(a_1 + a_n).$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Using this formula, find the sum of the arithmetic sequence.

-7, -3, 1, 5, 9, 13 ⓲

$$6\left(\frac{-7 + 13}{2}\right) = 18$$

$$\sum_{n=1}^{6} -7 + (n-1)4 \cdot$$

The sum of a sequence is called a series.

In the expression $\displaystyle\sum_{i=1}^{n} a_i$

i represents the term number (or index of summation), and a_i represents the general term of the sequence being summed

So therefore, $\displaystyle\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + ... + a_n$

Example find $\displaystyle\sum_{n=1}^{3} 2n + 1$

$$n \quad \frac{1 \quad 2 \quad 3}{3 + 5 + 7}$$

$$\longrightarrow 15$$

Example find

$$\sum_{n=1}^{3} n^3 \qquad\qquad \sum_{n=1}^{5} n^2 - n$$

Write in summation form the following series

1. -3, 3, 9, ... 111

1 2 3 4 5 6

$$\sum_{n=1}^{20} -3 + (n-1)6$$

2. 4, -1, -6, ⁴⁻ᵗ⁴-21

$$\sum_{n=1}^{6} 4 + (n-1) - 5$$

$a_n = 4 + (n-1) - 5$
$a_n = 4 + -5n + 5$
$-21 = 4 + -5n + 5$
$-21 = -5n + 9$

$a_n = a_1 + (n-1)d$
$111 = -3 + (n-1)6$
$111 = -3 + 6n - 6$
$111 = 6n - 9$
$\frac{+9}{120} = 6n$
$120 = 6n$
$\frac{120}{6} = n$

In Exercises 21–24, the sequences are arithmetic. Find

(a) the common difference,

(b) the tenth term,

(c) a recursive rule for the nth term, and

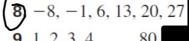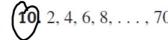(d) an explicit rule for the nth term.

P. $\frac{739}{21 - 24}$, 29

21. 6, 10, 14, 18, ...
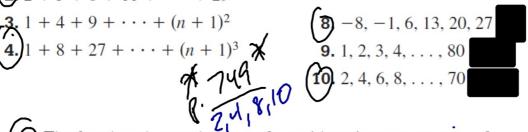
22. −4, 1, 6, 11, ...

23. −5, −2, 1, 4, ...

24. −7, 4, 15, 26, ...

In Exercises 1–6, write each sum using summation notation, assuming the suggested pattern continues.

2. 2 + 5 + 8 + 11 + · · · + 29

3. 1 + 4 + 9 + · · · + (n + 1)²

4. 1 + 8 + 27 + · · · + (n + 1)³ ✗

✗ 749
P. 2,4, 8, 10

In Exercises 7–12, find the sum of the arithmetic se

8. −8, −1, 6, 13, 20, 27 ■

9. 1, 2, 3, 4, . . . , 80 ■

10. 2, 4, 6, 8, . . . , 70 ■

29. The fourth and seventh terms of an arithmetic sequence are −8 and 4, respectively. Find the first term and a recursive rule for the nth term.