Warm up

Simplify the following radicals:

1. 128
2. 147

Find the distance between two points.

3. (0,4) and (3,5)
4. (2,1) and (-4,8)

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

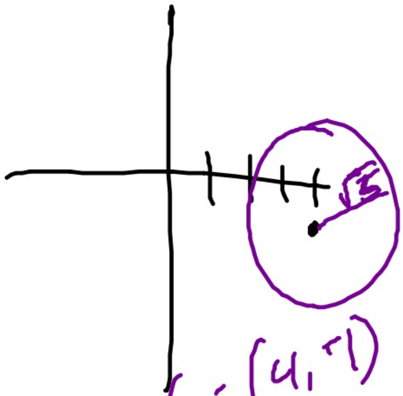
$\sqrt{9+1}$

5. Complete the square: $x^2 + y^2 - 8x + 2y + 12 = 0$

Warm up

1. $8\sqrt{2}$
2. $7\sqrt{3}$

3. $\sqrt{10}$
4. $\cancel{4}\sqrt{5}$ $\sqrt{85}$



$C = (4, -1)$

5. $x^2 - 8x + 16 + y^2 + 2y + 1 = -12 + 16 + 1$

$(x - 4)^2 + (y + 1)^2 = 5$

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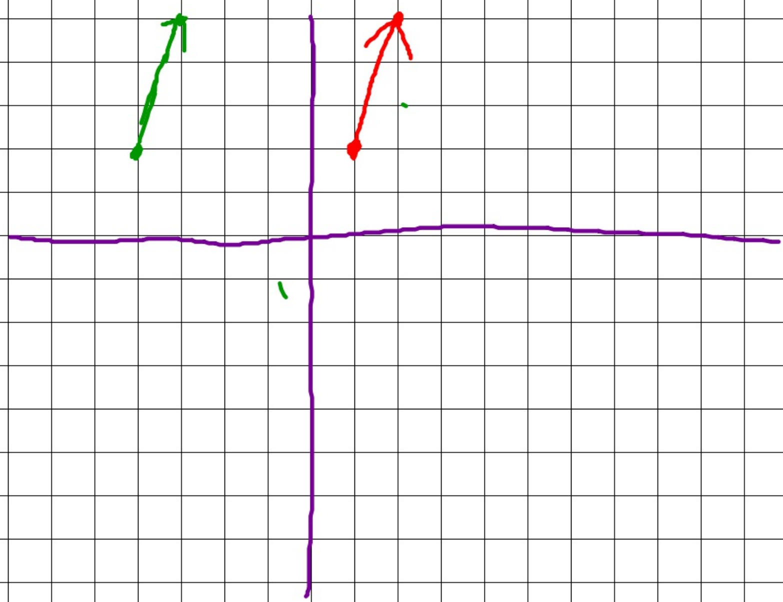
Vectors

Objective:

Vectors are directed line segments

Vectors have an initial point and a terminal point

Example $\overrightarrow{PQ}$ starts at point P (1, 2) and terminates at Q (2, 5)



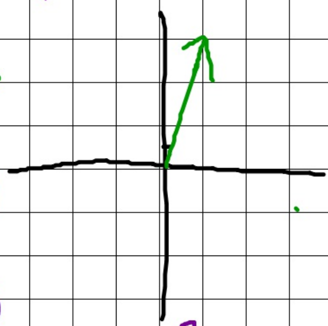
Component form has an initial point at the origin.

$$< x_2 - x_1, y_2 - y_1 >$$

Find component form of P (1,2) and Q (2,5)

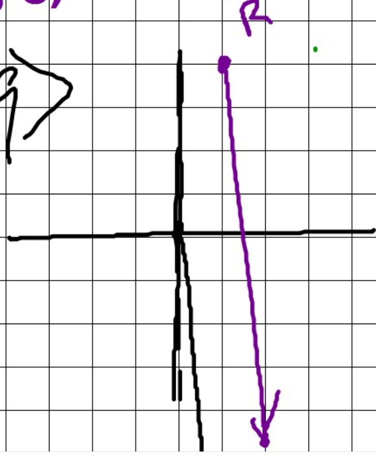
$V = \langle 1, 3 \rangle$

$(0,0) \rightarrow (1,3)$



Component form of R (1,4) and S (2,-5)

$\langle 1, -9 \rangle$



Addition and scalars of vectors

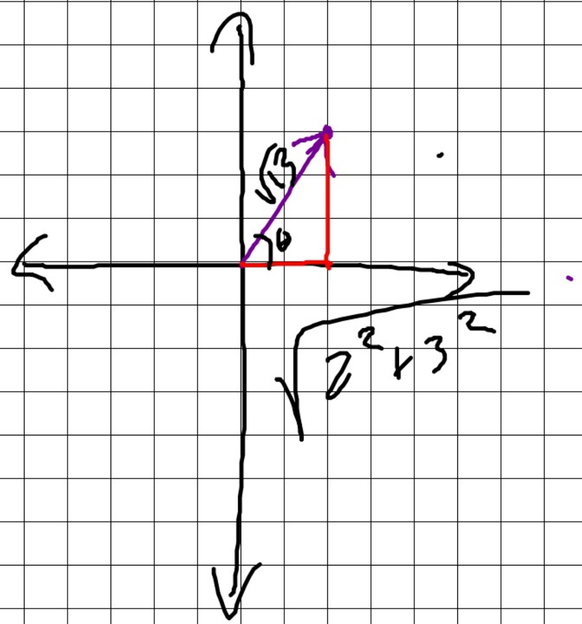
u = <-3, 4> v = <-2, 3> Find the following

⟨-6,8⟩ ⟨-4,6⟩ ⟨-10,14⟩ ⟨-13,18⟩

1. u+v 2. 2u 3. 2v 4. 2u+2v 5. 3u+2v

Magnitude of a vector is the distance between the two points.

v = <2, 3>



Two vectors are equivalent if the lengths are the same and they point in the same direction

Find the component form, magnitude, and determine if the following vectors are equivalent.

$\overrightarrow{RS}$ **R = (2, 1) S = (0, -1)** $\overrightarrow{PQ}$ **P = (1, 4) Q = (-1, 2)**

$v = \langle -2, -2 \rangle$

$\|v\| = 2\sqrt{2}$

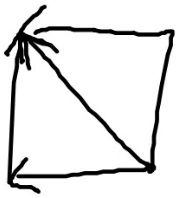
$|v|$

$u = \langle -2, -2 \rangle$

$\|u\|\ 2\sqrt{2}$

$|u|$

Vectors



Unit vector is a vector in the same direction that has a magnitude of 1

$$u = \frac{v}{\|v\|}$$

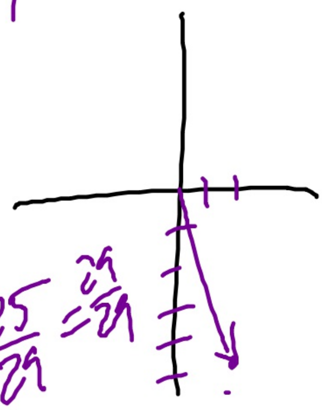
$\left\langle \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}} \right\rangle$

Find the unit vector for <2,-5>

$\|v\|$

$\sqrt{29}$

$\frac{4}{29} + \frac{25}{29} = \frac{29}{29}$



Unit vector is a vector in the same direction that has a magnitude of 1

$$u = \frac{v}{\|v\|}$$

Find the unit vector for <3, -1>

$\sqrt{10}$ $\left\langle \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle$

Find the unit vector for <-2, 6>

$\frac{-\cancel{2}}{\cancel{2}\sqrt{10}}$ $\sqrt{40}$ $2\sqrt{10}$ $\left\langle \frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$ $\frac{4}{\sqrt{80}}$

Find the unit vector for <4, -8>

$\sqrt{80}$ $\left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$

Find a vector in the direction of u = <-2, 4> that has a magnitude of 5.

$$\left\langle \frac{-2}{2\sqrt{5}}, \frac{4}{2\sqrt{5}} \right\rangle$$

$$5\left\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$\boxed{\left\langle \frac{-5}{\sqrt{5}}, \frac{10}{\sqrt{5}} \right\rangle}$$

$$\frac{-5\sqrt{5}}{5}, \frac{10\sqrt{5}}{5}$$

$$\langle -\sqrt{5}, 2\sqrt{5} \rangle$$

Unit vector <1,0> and <0,1> are called standard unit vectors. i=<1,0> and j=<0,1> Any vector can be written as a linear combination with the standard unit vector

Example <2, 5> can be written as 2i + 5j

Component form

$2\langle 1,0\rangle + 5\langle 0,1\rangle$

$\langle 2,0\rangle + \langle 0,5\rangle$

$f \circ g(x)$

$f(g(x))$

$\langle 2,5\rangle$ $2i + 5j$

Wrap up

Find the component form, magnitude, and unit vector for the following

1. $v=\langle 2,5\rangle$
2. $u=-2i+4j$
3. $w= -4j$

Find

4. $v+u$ 5. $3v+2w$ 6. $u+w$ 7. $v \cdot w$
8. $u \cdot w+v$

that they represent the same vector.

1. $R = (-4, 7)$, $S = (-1, 5)$, $O = (0, 0)$, and $P = (3, -2)$

2. $R = (7, -3)$, $S = (4, -5)$, $O = (0, 0)$, and $P = (-3, -2)$

3. $R = (2, 1)$, $S = (0, -1)$, $O = (1, 4)$, and $P = (-1, 2)$

4. $R = (-2, -1)$, $S = (2, 4)$, $O = (-3, -1)$, and $P = (1, 4)$

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In Exercises 5–12, let $P = (-2, 2)$, $Q = (3, 4)$, $R = (-2, 5)$, and $S = (2, -8)$. Find the component form and magnitude of the vector.

5. $\overrightarrow{PQ}$ [redacted]

6. $\overrightarrow{RS}$ [redacted]

7. $\overrightarrow{QR}$

8. $\overrightarrow{PS}$

9. $2\overrightarrow{QS}$ $\langle -2, -24\rangle$; $2\sqrt{145}$

10. $(\sqrt{2})\overrightarrow{PR}$ $\langle 0, 3\sqrt{2}\rangle$; $3\sqrt{2}$

11. $3\overrightarrow{QR} + \overrightarrow{PS}$ $\langle -11, -7\rangle$; $\sqrt{170}$

12. $\overrightarrow{PS} - 3\overrightarrow{PQ}$ $\langle -11, -16\rangle$; $\sqrt{377}$

In Exercises 13–20, let $\mathbf{u} = \langle -1, 3\rangle$, $\mathbf{v} = \langle 2, 4\rangle$, and $\mathbf{w} = \langle 2, -5\rangle$. Find the component form of the vector.

13. $\mathbf{u} + \mathbf{v}$ [redacted]

14. $\mathbf{u} + (-1)\mathbf{v}$ $\langle -3, -1\rangle$

15. $\mathbf{u} - \mathbf{w}$

16. $3\mathbf{v}$ $\langle 6, 12\rangle$

17. $2\mathbf{u} + 3\mathbf{w}$ [redacted]

18. $2\mathbf{u} - 4\mathbf{v}$ [redacted]

19. $-2\mathbf{u} - 3\mathbf{v}$

20. $-\mathbf{u} - \mathbf{v}$

In Exercises 21–24, find a unit vector in the direction of the given vector.

21. $\mathbf{u} = \langle -2, 4 \rangle$

22. $\mathbf{v} = \langle 1, -1 \rangle$

23. $\mathbf{w} = -\mathbf{i} - 2\mathbf{j}$

24. $\mathbf{w} = 5\mathbf{i} + 5\mathbf{j}$

In Exercises 25–28, find the unit vector in the direction of the given vector. Write your answer in **(a)** component form and **(b)** as a linear combination of the standard unit vectors **i** and **j**.

25. $\mathbf{u} = \langle 2, 1 \rangle$

26. $\mathbf{u} = \langle -3, 2 \rangle$

27. $\mathbf{u} = \langle -4, -5 \rangle$

28. $\mathbf{u} = \langle 3, -4 \rangle$

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Dot product of the vectors

DEFINITION Dot Product

The **dot product** or **inner product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.$$

Dot product of the vectors

Properties of the Dot Product

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$
3. $\mathbf{0} \cdot \mathbf{u} = 0$
4. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

   $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$
5. $(c\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$