Warm-up

Factor and simplify the following (if necessary):

1. $12x^2 + x - 6$

2. $16x^3 - 4x$

3. $\dfrac{x - 2}{x^2 + 4x - 12}$

4. $\dfrac{2x^2 + 17x + 30}{x^2 + 3x - 18}$

Graphing Rational Functions

OBJECTIVES:

1) Find the horizontal asymptotes, vertical asymptotes, and holes.

2) Find Slant asymptotes.

3) Graph rational functions and give the domain and range.

4) Describe end behavior

Given $f(x) = \frac{g(x)}{h(x)}$ Factor g(x) and h(x), then cancel common factors.

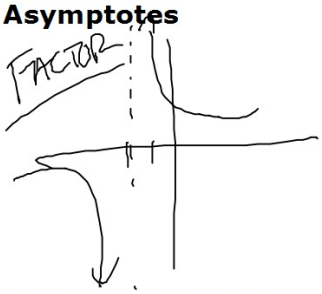
$y = \frac{x-5}{x^2-3x-10}$ ORIGINAL

$y = \frac{x-5}{(x-5)(x+2)} = \frac{1}{x+2}$

$x+2=0$ ← FACTOR

$x=-2$

Vertical Asymptotes



Holes (point discontinuity)

ORIGINAL

COORDINATE POINT $x-5=0$

$x=5$

$(5, \frac{1}{7})$

Horizontal Asymptotes

$f(x) = \dfrac{\text{smaller degree}}{\text{larger degree}}$

$y = 0$

$f(x) = \dfrac{\text{same degree}}{\text{same degree}}$

$\dfrac{\text{lead coeficient of numerator}}{\text{lead coeficient of denominator}}$

$f(x) = \dfrac{\text{larger degree}}{\text{smaller degree}}$

none

$\frac{1}{x}$ $\frac{x}{x^2+1}$ $\frac{2x}{3x^3}$ $\frac{3x^3+2}{2x^2}$ $\frac{2x^2+3}{3x^2-}$ $y = 2/3$

Slant Asymptotes

When the top is 1 degree higher than the bottom.

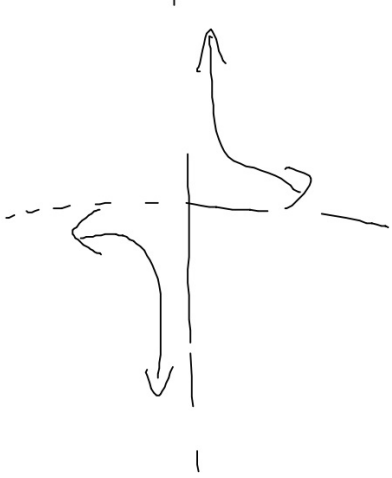
Get slant asymptote by long division.

$\frac{x^2+3x+13}{x+1}$

$x+1 \overline{)x^2+3x+13}$

GUIDED PRACTICE:
Determine the equations of any vertical asymptotes, horizontal asymptotes, and the coordinates (x, y) for any holes in the graph.

1) $f(x) = \dfrac{1}{x-1}$



VA:

Holes:

HA:

SA:

End Behavior:

Domain:
Range:

$\lim_{x \to \infty} f(x) = 0$

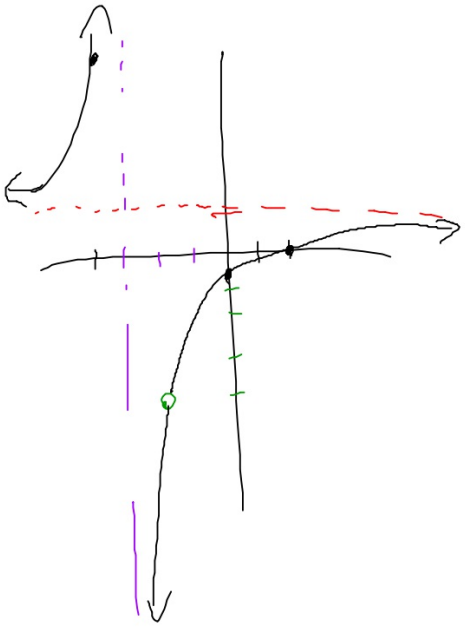
$\lim_{x \to -\infty} f(x) = 0$

D: $(-\infty, 1) \cup (1, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

2) $f(x) = \dfrac{x^2 - 4}{x^2 + 5x + 6}$

$\dfrac{(x+2)(x-2)}{(x+2)(x+3)}$ $\quad y = \dfrac{x-2}{x+3}$ $\quad \dfrac{-4}{1}$



VA: $x = -3$

Holes: $(-2, -4)$ $\quad x+2=0$, $x=-2$

HA: $y = \frac{1}{1}$ $\quad y = 1$

SA: ——

End Behavior: $\lim_{x \to \pm\infty} f(x) = 1$

Domain: $(-\infty, -3) \cup (-3, -2) \cup (-2, 2) \cup (2, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

$x\text{-int} = 2$

$y\text{-int} = -\frac{2}{3}$

$\dfrac{-4-2}{-4+3} = \dfrac{-6}{-1}$

3) $f(x) = \dfrac{x^2 - x}{x - 1}$

VA:

Holes:

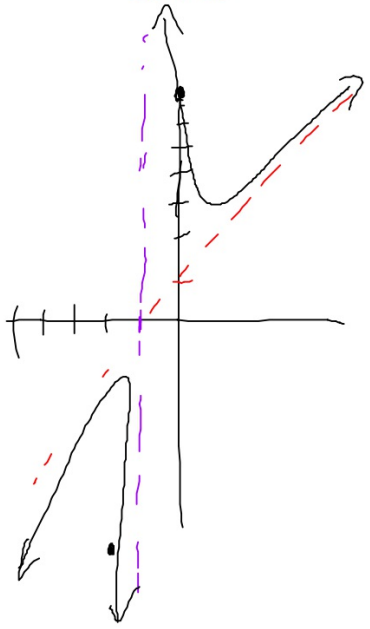
HA:

SA:

End Behavior:

Domain:
Range:

4) $\dfrac{x^2+2x+7}{2x+1} = \dfrac{7}{-1}$



VA: $x=-1$

Holes: —

HA: —

SA: $y=x+1$

End Behavior:

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

Domain:

Range:

x-int = ~~DNE~~

y-int. = 7

$x+1 \overline{)\, x^2+2x+7}$

$-(x^2+x)$

$x+7$

$x+1$

0

$$f(x)=\frac{x^2-4}{x-2}$$

$$f(x)=\frac{x}{x+1}$$

VA:

Holes:

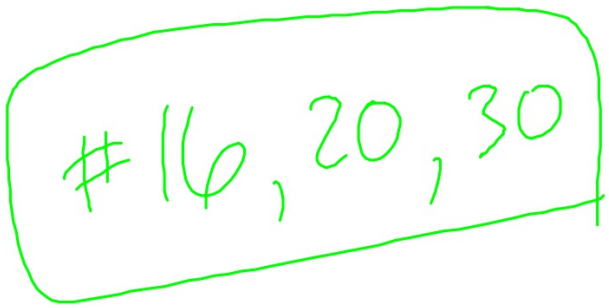
HA:

End Behavior:

Domain:
Range:

$$f(x)=\frac{3x^2-2}{x^2+5x+4}$$

$$f(x)=\frac{2x-3}{x-2}$$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