Warm up

1. What is the 100th term of the following sequence?
-2, 5, 12, 19, 26 ....

691

a_n
$23 = a_1 + (3)(7)$

2. An arithmetic sequence has a fourth term of 23 and a 9th term of 58. Find an explicit formula for the sequence.

$\frac{35}{5} =$ $a_n = 2 + (n-1)7$

3. What is the ratio of each Fibonacci number to the number preceding: i.e. 8/5, 13/8, 21/13 .... ?
What happens to the ratio as the numbers get larger?

ϕ 1.618

$a_n = 7n - 5$

21. (a) $d = 4$
    (b) $a_{10} = 6 + 9(4) = 42$
    (c) Recursive rule: $a_1 = 6$; $a_n = a_{n-1} + 4$ for $n \geq 2$
    (d) Explicit rule: $a_n = 6 + 4(n - 1)$
22. (a) $d = 5$
    (b) $a_{10} = -4 + 9(5) = 41$
    (c) Recursive rule: $a_1 = -4$; $a_n = a_{n-1} + 5$ for $n \geq 2$
    (d) Explicit rule: $a_n = -4 + 5(n - 1)$
23. (a) $d = 3$
    (b) $a_{10} = -5 + 9(3) = 22$
    (c) Recursive rule: $a_1 = -5$; $a_n = a_{n-1} + 3$ for $n \geq 2$
    (d) Explicit rule: $a_n = -5 + 3(n - 1)$
24. (a) $d = 11$
    (b) $a_{10} = -7 + 9(11) = 92$
    (c) Recursive rule: $a_1 = -7$; $a_n = a_{n-1} + 11$ for $n \geq 2$
    (d) Explicit rule: [illegible]

29. $a_4 = -8 = a_1 + 3d$ and $a_7 = 4 = a_1 + 6d$, so $a_7 - a_4 = 12 = 3d$. Therefore $d = 4$, so $a_1 = -8 - 3d = -20$ and $a_n = a_{n-1} + 4$ for $n \geq 2$.

30. $a_5 = -5 = a_1 + 4d$ and $a_9 = -17 = a_1 + 8d$, so $a_9 - a_5 = -12 = 4d$. Therefore $d = -3$, so $a_1 = -5 - 4d = 7$ and $a_n = a_{n-1} - 3$ for $n \geq 2$.

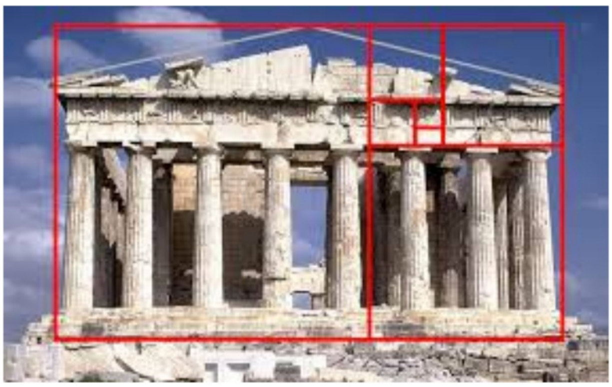
1, 1, 2, 3, 5, 8, 13

2. $\sum_{k=1}^{10} (3k - 1)$

4. $\sum_{k=1}^{n+1} k^3$

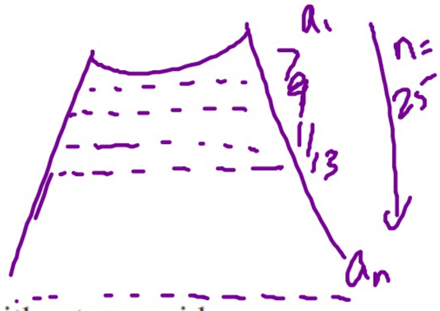
8. $6 \cdot \left(\frac{-8 + 27}{2}\right) = 3 \cdot 19 = 57$

10. $35 \cdot \left(\frac{2 + 70}{2}\right) = 35 \cdot 36 = 1260$



39. Arena Seating The first row of seating in section J of the Athena Arena has 7 seats. In all, there are 25 rows of seats in section J, each row containing two more seats than the row preceding it. How many seats are in section J? 775

$$\sum_{n=1}^{25} 7+(n-1)2$$



40. Patio Construction Pat designs a patio with a trapezoid-shaped deck consisting of 16 rows of congruent slate tiles. The numbers of tiles in the rows form an arithmetic sequence. The first row contains 15 tiles and the last row contains 30 tiles. How many tiles are used in the deck? 360

$$\sum_{n=1}^{16} 15+(n-1)1$$

Objective: Identify geometric sequences and series.

2, 4, 8, 16, 32, 64 ...

8, 4, 2, 1 ...

Notation:

a_1 **means**

a_n **means**

r means RATIO

n	1	2	3	4	5	6	7
a_n	a_1						
SEQ	64	32	16	8	4	2	1
	2	-4	8	-16	32	-64	

a_n

$r = 1/2$

$r = 2$.

Geometric sequences are sequences where a common ratio is multiplied to create the next term.

A geometric sequence looks like this:

$a_1, a_1r, a_1r^2, a_1r^3, \ldots, a_1r^{n-1}$

Explicit formula is: $a_n = a_1 \cdot r^{(n-1)}$

to find the common ratio:

$r = \frac{a_2}{a_1}$

Ex For each geometric sequence below

a) find the common ratio

b) find an explicit rule for the nth term

~~c) find a recursive rule for the nth term~~

d) find the 10th term

1. 3, 6, 12, 24, 48...

$r=2$

$a_n = 3 \cdot 2^{n-1}$

$a_{10} = 1536$

2. 1, -2, 4, -8, 16...

a) $r=-2$

b) $a_n = 1 \cdot -2^{(n-1)}$

d) $a_{10} = -512$

$r = \frac{-2}{1}$ $r=-2$

Find the nth term of the geometric sequence.

~~1. $a_1 = 4$, $r = ½$, $n = 10$~~

$a_6 = -\frac{2}{9}$

$a_n = a_1 r^{n-1}$

2. $a_2 = -18$, $a_5 = 2/3$, $n = 6$ 6TH TERM?

$-18 = a_1 \cdot r^1$

$2/3 = a_1 \cdot r^4$

$\frac{-18}{r} = \frac{2}{3r^4}$

$-54r^4 = 2r$

$-54r^3 = 2$

$r^3 = \frac{2}{-54}$

$r = -1/3$

$\frac{-18}{r} = a_1$

$\frac{2/3}{r^4} = a_1$

Given two terms in a geometric sequence find the 8

24) $a_5 = 768$ and $a_2 = 12$

$a_8 = 49152$

$768 = a_1 \cdot r^4$

$12 = a_1 r^1$

$\frac{768}{r^4} = \frac{12}{r^1}$

$768 r = 12 r^4$

$r = 4$

26) $a_5 = 3888$ and $a_3 = 108$ 12

$a_8 = 839808$

$a_1 \cdot \frac{4}{4} = \frac{a_2}{4}$

$a_1 = 3$

$3 \cdot 4^7$

Finding the Sum of a Geometric Series

$a_n = a_1 r^{n-1}$

Then the sum of the terms of the sequence is

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

$$\boxed{= \frac{a_1(1 - r^n)}{1 - r}}$$

$n-1 = 10$
$+1 \quad +1$
$n = 11$

EXAMPLE 2 **Summing the Terms of a Geometric Sequence**

Find the sum of the geometric sequence 4, −4/3, 4/9, −4/27, . . . , $4(-1/3)^{10}$.

a_1

$$\frac{4\left(1-\left(-\frac{1}{3}\right)^{11}\right)}{1 - -\frac{1}{3}} = 3$$

$r = -\frac{1}{3}$

$n = 11$

Write the following series in summation form

1. -2, 4, -8, ..., 256 $\sum_{n=1}^{8} -2(-2)^{n-1} = \underline{\underline{170}}$

$a_n = a_1 r^{n-1}$

$256 = -2(-2)^{n-1}$

$-128 = (-2)^{n-1}$ $\ln 2$

$\ln 128 = n-1$

$7 = n-1$

$8 = n$

1 2 3 4 5 6

2. (1/2),(1/4),(1/8)......(1/64) $\sum_{n=1}^{6} \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$

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35. Savings Account The table below shows the December balance in a fixed-rate compound savings account each year from 1996 to 2000.

Year	1996	1997	1998	1999	2000
Balance	\$20,000	\$22,000	\$24,200	\$26,620	\$29,282

(a) The balances form a geometric sequence. What is r?

(b) Write a formula for the balance in the account n years after December 1996.

(c) Find the sum of the December balances from 1996 to 2006, inclusive.

(a) the common ratio,

(b) the eighth term,

(c) a recursive rule for the nth term, and

(d) an explicit rule for the nth term.

P. 739
25-28
31-32

P. 749
14, 15
5, 6

25. 2, 6, 18, 54, . . .

26. 3, 6, 12, 24, . . .

27. 1, -2, 4, -8, 16, . . .

28. -2, 2, -2, 2, . . .

31. The second and eighth terms of a geometric sequence are 3 and 192, respectively. Find the first term, common ratio, and an explicit rule for the nth term.

32. The third and sixth terms of a geometric sequence are -75 and -9375, respectively. Find the first term, common ratio, and an explicit rule for the nth term.

In Exercises 13–16, find the sum of the geometric sequence.

13. 3, 6, 12, . . . , 12,288

14. 5, 15, 45, . . . , 98,415

15. $42, 7, \frac{7}{6}, \ldots, 42\left(\frac{1}{6}\right)^8$

In Exercises 1–6, write each sum using summation not the suggested pattern continues.

5. $6 - 12 + 24 - 48 + \cdots$

6. $5 - 15 + 45 - 135 + \cdots$