Warm up

Find the missing sides or angles.

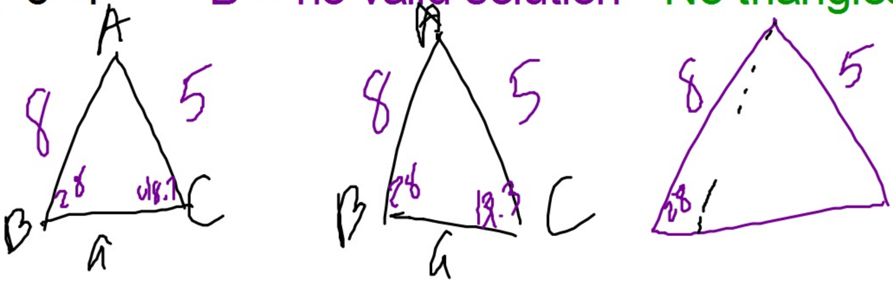
1. $a=7$ $A=37°$ $B=102°$ $b =$ ____________
2. $a=5$ $b=4$ $B= 105°$ $A =$ ____________
3. $A=38°$ $a=5$ $c=3$ $C =$ ____________
4. $B=28°$ $b=5$ $c=8$ $C =$ ____________
5. $C=47°$ $b=8$ $c=4$ $B =$ ____________

6. State the number of possible triangles in #1-5 above that can be formed for the given measurements in each problem.

Warm up

Find the missing sides or angles.

1. a=7 A=37° B=102°   b = 11.38 - One triangle
2. a=5 b=4   B= 105°   A = no valid solution - No triangles
3. A=38° a=5 c=3   C = 21.69° - One Triangle
4. B=28° b=5 c=8   C = 48.7° or 131.3° - Two triangles
5. C=47° b=8 c=4   B = no valid solution - No triangles



2. Given: $c = 17, B = 15°, C = 120°$ — an AAS case.

$$A = 180° - (B + C) = 45°;$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow a = \frac{c \sin A}{\sin C} = \frac{17 \sin 45°}{\sin 120°} \approx 13.9$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow b = \frac{c \sin B}{\sin C} = \frac{17 \sin 15°}{\sin 120°} \approx 5.1$$

5. Given: $A = 40°, B = 30°, b = 10$ — an AAS case.

$$C = 180° - (A + B) = 110°;$$

$$a = \frac{b \sin A}{\sin B} = \frac{10 \sin 40°}{\sin 30°} \approx 12.9;$$

$$c = \frac{b \sin C}{\sin B} = \frac{10 \sin 110°}{\sin 30°} \approx 18.8$$

10. Given: $A = 49°, a = 32, b = 28$ — an SSA case.

$h = b \sin A \approx 21.1; h < b < a$, so there is one triangle.

$$B = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.660\ldots) \approx 41.3°$$

$$C = 180° - (A + B) = 89.7°;$$

$$c = \frac{a \sin C}{\sin A} = \frac{32 \sin 89.7°}{\sin 49°} \approx 42.4$$

40. Given: $c = 2.32, A = 28°, B = 37°$ — an ASA cas

$$C = 180° - (A + B) = 115°;$$

$$a = \frac{c \sin A}{\sin C} = \frac{2.32 \sin 28°}{\sin 115°} \approx 1.2 \text{ mi.}$$

$$b = \frac{c \sin B}{\sin C} = \frac{2.32 \sin 37°}{\sin 115°} \approx 1.5 \text{ mi.}$$

Therefore, the altitude is $h = b \sin A \approx (1.5) \sin 2$
≈ 0.7 mi — or $a \sin B \approx (1.2) \sin 37°$ mi ≈ 0.7 mi.

20. $C_1 \approx 47.1°; A_1 \approx 94.9°; a_1 \approx 34.0; C_2 \approx 132.9°;$
$A_2 \approx 9.1°; a_2 \approx 5.4$

Solve each triangle. Round your answers to the nearest tenth.

More Practice

12) $m\angle B = 45°, a = 28, b = 27$

$m\angle C = 87.8°, m\angle A = 47.2°, c = 38.2$

Or $m\angle C = 2.2°, m\angle A = 132.8°, c = 1.5$

[redacted]

13) $m\angle C = 145°, b = 7, c = 33$

$m\angle A = 28°, m\angle B = 7°, a = 27$

14) $m\angle B = 73°, a = 7, b = 5$

Not a triangle

Objective: To solve triangles using the law of Cosines

Law of Cosines is used when we have SAS or SSS

Law of Cosines

Let $\triangle ABC$ be any triangle with sides and angles labeled in the usual way (Figure 5.22).

Then

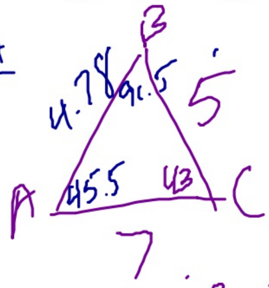
$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

If given SAS after you find the missing side use law of sines to find the remaining sides.

Example 3.   a=5   b=7   C= 43°

$$\frac{\sin 43}{4.78} \qquad \frac{\sin B}{7}$$

$$\frac{\sin 43}{4.78} = \frac{\sin A}{5}$$



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 5^2 + 7^2 - 2(5)(7)\cos(43)$$

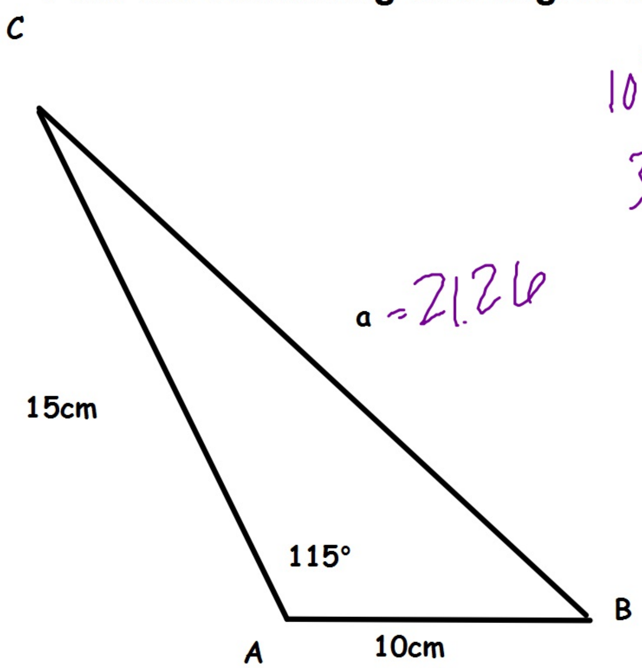
$$c^2 = 74 - 51.19$$

$$\sqrt{c^2} = \sqrt{22.8}$$

$$c = 4.78$$

Example 4: Two sides and the included angle SAS

Find the remaining two angles and the side of the triangle



$$10^2+15^2-2(10)(15)\cos 115$$

$$325 - -126.78$$

$$\sqrt{451.78}$$

- If given SSS one must find the largest angle first.
- If given SSS the sum of the two smallest sides must be larger than the length of the longest side.
- After you find that angle then use the law of sines to find the remaining angles.

Rearrange the formula to have the cosine of an angle by itself; if using SSS.

Example 1. a=5 b=12 c=15

$15^2 = 5^2 + 12^2 - 2(5)(12)\cos C$

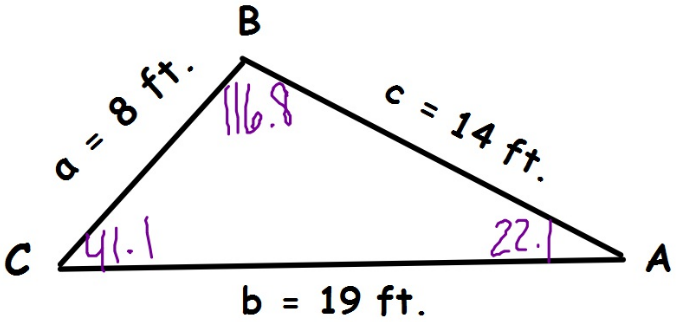
$225 = 169 - 120\cos C$

$56 = -120\cos C$

117.81°

Example 2: Three sides of a Triangle SSS

Find the 3 angles of the triangle



Solve each triangle. Round your answers to the nearest tenth.

19) In ΔABC, $a = 14$ cm, $b = 9$ cm, $c = 6$ cm

21) In ΔQRP, $q = 12$ in, $p = 28$ in, $r = 18$ in

20) In ΔXYZ, $m\angle X = 138°$, $y = 15$ in, $z = 25$ in

22) In ΔQRP, $p = 28$ km, $q = 17$ km, $r = 15$ km

Example 5: To approximate the length of a marsh, a surveyor walks 250m from point A to point B, then turns and walks 220m to point C. The m<B = 105°. Find the length of AC.

The same parts that determine a triangle also determine its area. If the parts happen to be two sides and an included angle (SAS), we get a simple area formula in terms of those three parts that does not require finding an altitude.

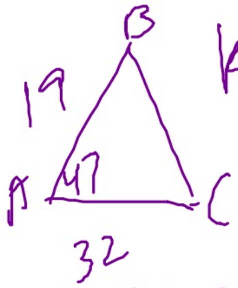
Area of a Triangle

$$\triangle\ \overset{K}{\text{Area}} = \boxed{\frac{1}{2}bc \sin A} = \boxed{\frac{1}{2}ac \sin B} = \boxed{\frac{1}{2}ab \sin C}$$

In Exercises 17–20, find the area of the triangle.

17. $A = 47°, \quad b = 32 \text{ ft}, \quad c = 19 \text{ ft}$

$K = 222.3 \text{ ft}^2$

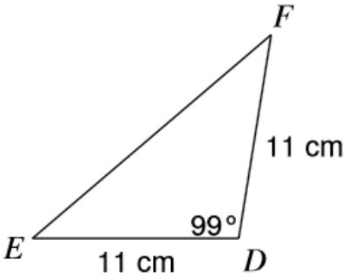


$K = \frac{1}{2} bc \sin A$

$K = \frac{1}{2}(32)(19)\sin(47)$

Find the area of each triangle to the nearest tenth.

25)



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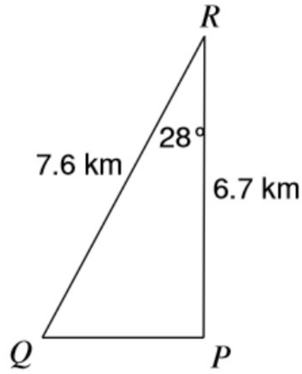
26)



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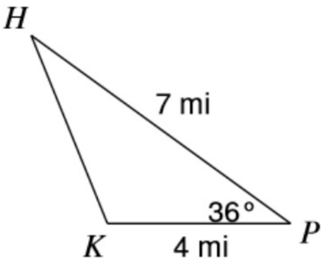
27)



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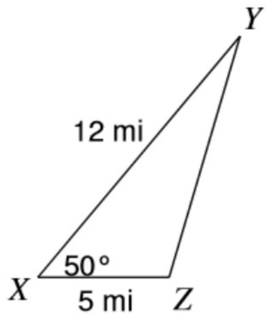
28)



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There is also an area formula that can be used when the three sides of the triangle are known.

THEOREM **Heron's Formula**

Let a, b, and c be the sides of $\triangle ABC$, and let s denote the **semiperimeter**

$$(a + b + c)/2.$$

Then the area of $\triangle ABC$ is given by Area $= \sqrt{s(s-a)(s-b)(s-c)}$.

Example 8: a=5, b=9, and c=10. Find the area of the triangle.

$\frac{24}{2}$ $s = 12$

$\sqrt{12(12-5)(12-9)(12-10)}$

$\sqrt{504}$ ft^2