Warm-up

Graph the following rational functions, and list the VA, HA, holes and SA if possible. Also list the range and domain of each.

1. $f(x) = \frac{4x-5}{1-2x}$

1-2x

2. $f(x) = \frac{x^2-4x-5}{x-5}$

$(x-5)(x+1)$

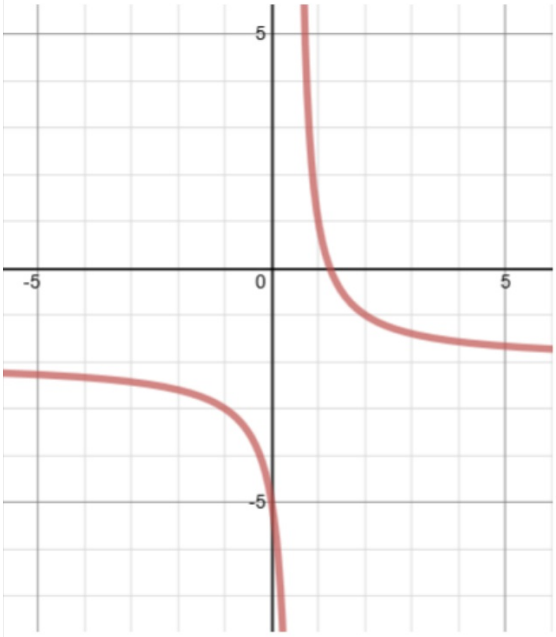
3. $f(x) = \frac{4x+2}{x^2-4}$

$2(2x+1)$

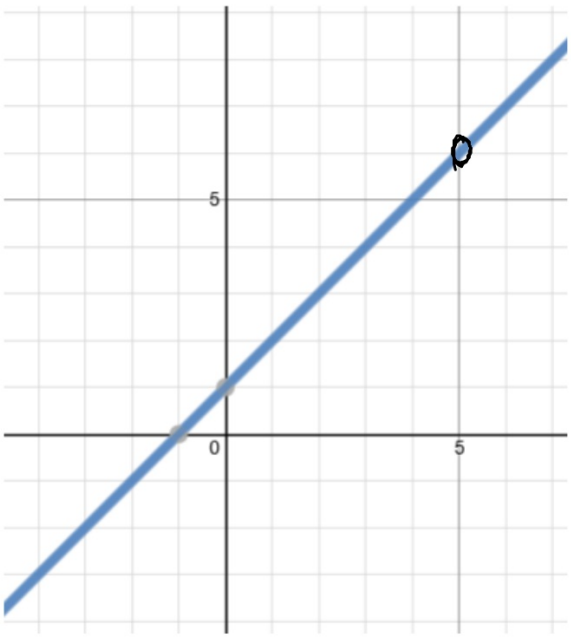
$(x+2)(x-2)$

	VA	HA	SA	HOLE	DOMAIN
1.	$\frac{1}{2}$	-2	—	—	$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$
2.	—	—	—	$(5, 6)$	$(-\infty, 5) \cup (5, \infty)$
3.	± 2	0	—	—	$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

$$f(x) = \frac{4x-5}{1-2x}$$

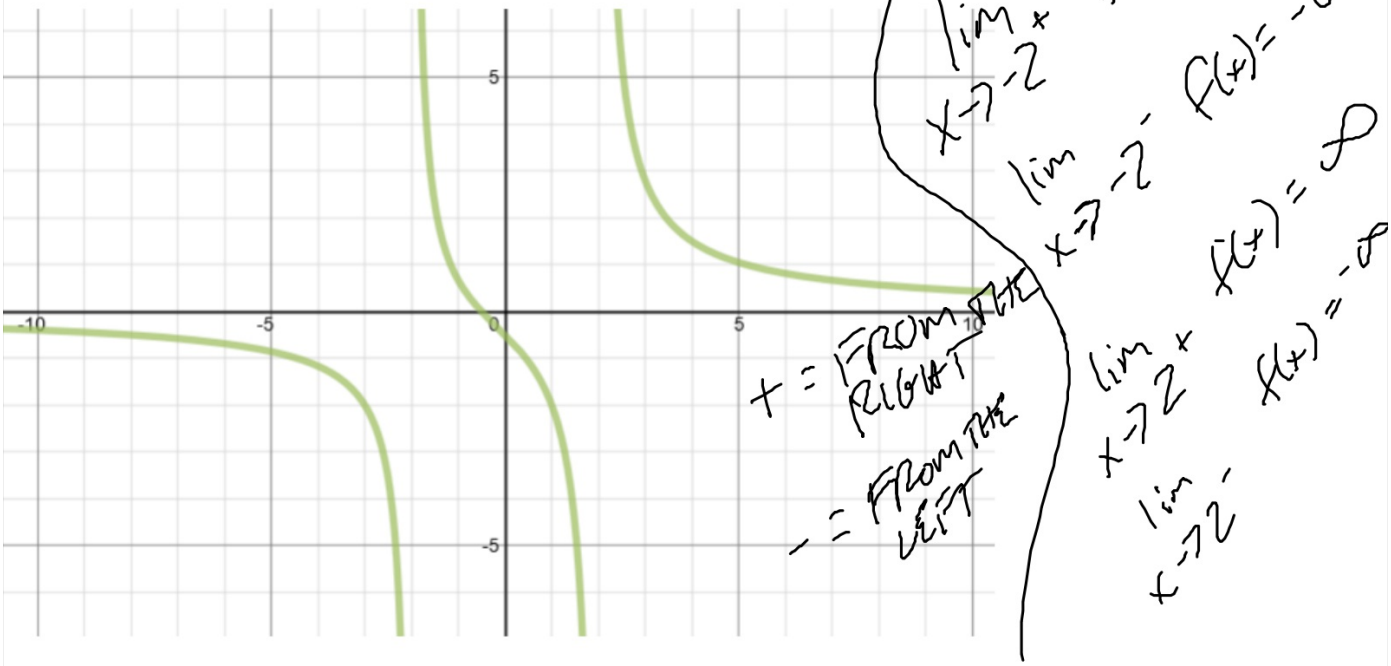


$$f(x) = \frac{x^2-4x-5}{x-5}$$



$$f(x) = \frac{4x+2}{x^2-4}$$

Limits Notes...



$\lim_{x \to -2^+} f(x) = \infty$

$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = -\infty$

+ = FROM THE RIGHT

− = FROM THE LEFT

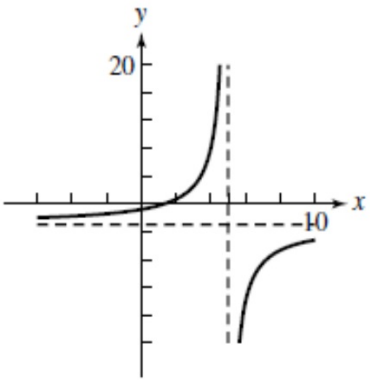
Exercise Set 2.3 Worksheet
(Back of Rational functions cheat sheet)

#16, 20, 30

2. Domain: all $x \neq 1$; $\lim_{x \to 1^-} f(x) = \infty$; $\lim_{x \to 1^+} f(x) = -\infty$

4. Domain: all $x \neq -1, 1$; $\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to 1^-} f(x) = -\infty$, $\lim_{x \to 1^+} f(x) = \infty$

10. Translate right 5 units, vertically stretch by 11, reflect across x-axis, translate down 3 units.
Asymptotes: $x = 5$, $y = -3$



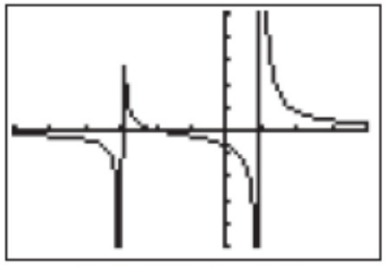
19. Vertical asymptote: none; Horizontal asymptote: $y = 2$; $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 2$

22. Vertical asymptotes: $x = -3, x = 0$; Horizontal asymptote: $y = 0$;
$\lim_{x \to 0^+} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$

$$\lim_{x \to -3^-} f(x) = -\infty, \lim_{x \to -3^+} f(x) = \infty, \lim_{x \to 0^-} f(x) = \infty,$$

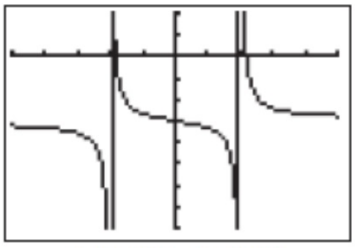
24. Intercepts: $\left(0, -\frac{2}{3}\right)$ and $(-2, 0)$

Asymptotes: $x = -3$, $x = 1$, and $y = 0$



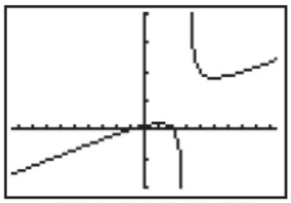
$[-6, 4]$ by $[-5, 5]$

28. Intercepts: $(0, -3)$, $(-1.84, 0)$, and $(2.17, 0)$; Asymptotes: $x = -2$, $x = 2$, and $y = -3$



$[-5, 5]$ by $[-8, 2]$

44. Intercepts: $(-1, 0)$, $(2, 0)$, $\left(0, \frac{2}{3}\right)$; asymptotes: $x = 3$, $y = x + 2$; $\lim_{x \to 3^-} k(x) = -\infty$, $\lim_{x \to 3^+} k(x) = \infty$



[–9.4, 9.4] by [–10, 20]

Domain: $x \neq 3$; Range: $(-\infty, 1] \cup [9, \infty)$;
Continuity: all $x \neq 3$;
Increasing: $(-\infty, 1]$, $[5, \infty)$; Decreasing: $[1, 3)$, $(3, 5]$;
No symmetry; Unbounded; Local max at $(1, 1)$, local min at $(5, 9)$;
Horizontal asymptote: none; Vertical asymptote: $x = 3$;
Slant asymptote: $y = x + 2$; End behavior: $\lim_{x \to -\infty} k(x) = -\infty$, $\lim_{x \to \infty} k(x) = \infty$

Objective: Solve Rational Equations

How do you solve a rational equation?

STEPS

1. ***Factor denominators.***
2. ***Find the LCD of all denominators.***
3. ***Multiply every term on both sides of the equation by the LCD.***
4. ***Solve the resulting equation.***
5. ***CHECK in the original equation! We must check for excluded values, the values that make any denominator zero.***

lower the rectangle to reveal the steps

GUIDED PRACTICE:
Solve.
1.

$$\frac{x+1}{3x-6} = \frac{5}{6} + \frac{3}{x-2}$$

$3(x-2)$

LCD: $6(x-2)$

$$2 \cdot \frac{6(x-2)}{1} \cdot \frac{x+1}{3(x-2)}$$

$$2(x+1) = 5(x-2) + 18$$

$$2x+2 = 5x-10+18$$

$$2x+2 = 5x+8$$

$$-6 = 3x$$

$$-2 = x$$

2. $\frac{7}{r+2} = \frac{6}{r-5}$

$(r+2)(r-5)$ $(r+2)(r-5)$

LCD $(r+2)(r-5)$

$7(r-5) = 6(r+2)$

$7r - 35 = 6r + 12$

$r = 47$

3. $\dfrac{4}{x^2-2x-3} = \dfrac{x}{x-3} - \dfrac{1}{x+1}$

$(x-3)(x+1)$ $(x-3)(x+1)$ $(x-3)(x+1)$

LCD:

$4 = x(x+1) - (x-3)$

$4 = x^2 + x - x + 3$

$4 = x^2 + 3$

$-4 \quad -4$

$0 = x^2 - 1$

$x^2 = 1$

$x = \pm 1$

$x = 1$

-1 DOES NOT

5. $\frac{1}{x-3}+\frac{2}{x^2-9}=\frac{5}{x+3}$

$x+3+2=5(x-3)$

$x+5=5x-15$

$20=4x$

$5=x$

Practice problems:

1. $2 - \frac{1}{x+1} = \frac{1}{x^2+x}$

2. $x + \frac{4x}{x-3} = \frac{12}{x-3}$

3. $\frac{2}{x-1} + x = 5$

4. $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2+x-2}$

5. $\frac{3x}{x+5} + \frac{1}{x-2} = \frac{7}{x^2+3x-10}$

How do you solve a rational equation by graphing?

TI-83/84 STEPS

1. ***Y1= left side of the equation***
2. ***Y2= right side of the equation***
3. ***ZOOM 6: Standard***
4. ***2nd TRACE 5: Intersection***
5. ***Move the cursor as close as possible to the point of intersection and press ENTER 3 times. The x-value is the solution.***
6. ***Repeat steps 4-5 if another point of intersection exists.***

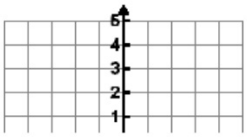
GUIDED PRACTICE:

LCD: $2(x-1)$

Solve by graphing.

6.

$$\frac{x}{x-1} - 1 = \frac{x}{2}$$



How do you solve a rational equation by graphing?

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z.

$$\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$$