Warm up

Finish Vector Worksheet #1
#13 - 28

Page 511 1,2,5,8,13,17,18,22,25,27,28

1. If $R = (-4, 7)$ and $S = (-1, 5)$, then, using the HMT rule,
$\overrightarrow{RS} = \langle -1 - (-4), 5 - 7 \rangle = \langle 3, -2 \rangle$.
If $P = (0, 0)$ and $Q = (3, -2)$, then, using the HMT rule,
$\overrightarrow{PQ} = \langle 3 - 0, -2 - 0 \rangle = \langle 3, -2 \rangle$.
Both vectors represent $\langle 3, -2 \rangle$ by the HMT rule.

2. If $R = (7, -3)$ and $S = (4, -5)$, then, using the HMT rule,
$\overrightarrow{RS} = \langle 4 - 7, -5 - (-3) \rangle = \langle -3, -2 \rangle$.
If $P = (0, 0)$ and $Q = (-3, -2)$, then, using the HMT rule,
$\overrightarrow{PQ} = \langle -3 - 0, -2 - 0 \rangle = \langle -3, -2 \rangle$.
Both vectors represent $\langle -3, -2 \rangle$ by the HMT rule.

5. $\overrightarrow{PQ} = \langle 3 - (-2), 4 - 2 \rangle = \langle 5, 2 \rangle$,
$|\overrightarrow{PQ}| = \sqrt{5^2 + 2^2} = \sqrt{29}$

8. $\overrightarrow{PS} = \langle 2 - (-2), -8 - 2 \rangle = \langle 4, -10 \rangle$,
$|\overrightarrow{PS}| = \sqrt{4^2 + (-10)^2} = \sqrt{116} = 2\sqrt{29}$

13. $\langle -1, 3\rangle + \langle 2, 4\rangle = \langle 1, 7\rangle$

17. $2\langle -1, 3\rangle + 3\langle 2, -5\rangle = \langle 4, -9\rangle$

18. $2\langle -1, 3\rangle - 4\langle 2, 4\rangle = \langle -10, -10\rangle$

22. $\dfrac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \dfrac{1}{\sqrt{1^2 + (-1)^2}}, \dfrac{-1}{\sqrt{1^2 + (-1)^2}} \right\rangle$
$\approx 0.71\mathbf{i} - 0.71\mathbf{j}$

25. **(a)** $\left\langle \dfrac{2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}} \right\rangle$

(b) $\dfrac{2}{\sqrt{5}}\mathbf{i} + \dfrac{1}{\sqrt{5}}\mathbf{j}$

27. **(a)** $\left\langle -\dfrac{4}{\sqrt{41}}, -\dfrac{5}{\sqrt{41}} \right\rangle$

(b) $-\dfrac{4}{\sqrt{41}}\mathbf{i} + \left(-\dfrac{5}{\sqrt{41}}\right)\mathbf{j} = -\dfrac{4}{\sqrt{41}}\mathbf{i} - \dfrac{5}{\sqrt{41}}\mathbf{j}$

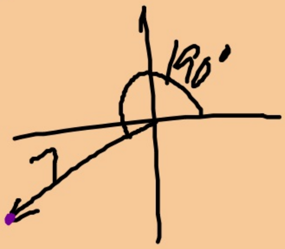
28. **(a)** $\left\langle \dfrac{3}{5}, -\dfrac{4}{5} \right\rangle$

(b) $\dfrac{3}{5}\mathbf{i} + \left(-\dfrac{4}{5}\right)\mathbf{j} = \dfrac{3}{5}\mathbf{i} - \dfrac{4}{5}\mathbf{j}.$

Objective: Find direction angles of vectors, angle between vectors, and vectors in the same direction with different magnitudes.

Directional angle is the angle the vector makes with the positive x axis

If v has a direction angle of θ the component of v is
$v = \langle \|v\| \cos\theta, \|v\| \sin\theta \rangle$



Find the component form of a vector that has a direction angle of 190° and a magnitude of 7.

$\langle 7\cos 190°, 7\sin 190° \rangle$

$\langle -6.9, -1.2 \rangle$

Find the component form of a vector that has:

1. Direction angle of 22° and a magnitude of 3.

$\langle 2.8, 1.1 \rangle$

2. Direction angle of 279° and a magnitude of 12.

$\langle 1.9, -11.9 \rangle$

Find the direction angle and magnitude of a vector:

v = <2, 3> $\|v\| = \sqrt{13}$

$2 = \sqrt{13}\cos\theta$ $\frac{2}{\sqrt{13}} = \cos\theta$

$\theta = 56.3^\circ$

b) v = <-2, 3> $\|v\| = \sqrt{13}$

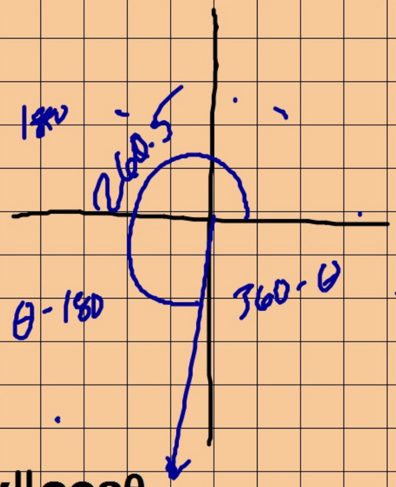
$\theta = 123.7$

c) v = <-1, -6> -6

$\|v\| = \sqrt{37}$

$\theta = 99.5$

80.5



We can use: $v_x = \|v\|\cos\theta$

Find the direction angle and magnitude of w = <-1,8>

$||w|| = \sqrt{65}$

$\theta = 97.1$

Find the direction angle and magnitude of v = <-2,4>

$||v|| = \sqrt{20}$

$\theta = 116.6$

Find the direction angle and magnitude of u = <-2,-4>

$||u|| = \sqrt{20}$

$\theta = 243.4$

What is the angle between u and v?

Dot product of the vectors

DEFINITION Dot Product

The **dot product** or **inner product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\boxed{\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.}$$

ONE NUMBER

Properties of the Dot Product

Let **u**, **v**, and **w** be vectors and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$
3. $\mathbf{0} \cdot \mathbf{u} = 0$
4. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

   $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$
5. $(c\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$

Find the dot product of u = <2, 3> and v = <-1, 6>

$2 \cdot -1 + 3 \cdot 6$

$-2 + 18$

$\boxed{16}$

Dot product of the vectors

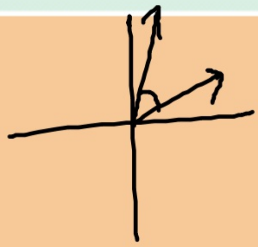
DEFINITION Dot Product

The **dot product** or **inner product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.$$

Find the dot product of:

ACUTE



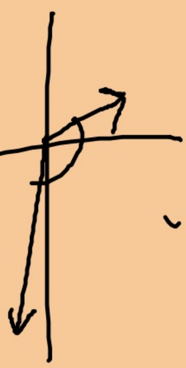
1) **u = <1, 7> and v = <5, 2>** 19

ORTHOGONAL

2) **u = <-2, 3> and v = <3, 2>** = 0

3) **u = <3, 1> and v = <-1, -7>** -10

OBTUSE

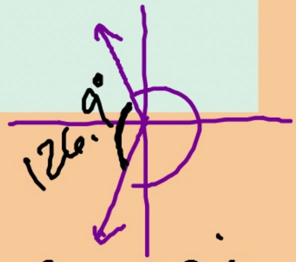


THEOREM Angle Between Two Vectors

If θ is the angle between the nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\boxed{\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}}$$

$$\text{and } \theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right).$$



Find the direction angle and magnitude of v = <-2,4>

$4 + -16$

Find the direction angle and magnitude of u = <-2,-4>

What is the angle between u and v?

$$\cos\theta = \frac{-12}{(\sqrt{20})(\sqrt{20})}.$$

What is the angle between u and v?

1) u = <2, 3> and v = <-2, 5> 55.5°

2) u = <2, 1> and v = <-1, -3> 135°

$$\cos\theta = \frac{-5}{(\sqrt{5})(\sqrt{10})}$$

Wrap-up

2 A child is pulling a sled through the snow with a force of 20 Newtons at an angle of 40°.

- To the nearest tenth, what is the vertical component of the force?
- To the nearest tenth, what is the horizontal component of the force?

$\langle 20\cos 40^\circ, 20\sin 40^\circ \rangle$

15.3 12.8

4 A wind that is blowing from the northwest toward the southeast can be represented by a vector. The vector has an eastward component and a southward component. If the eastward component has a magnitude of 5.00 miles per hour and the southward component has a magnitude of 15.00 miles per hour, in what direction is the wind blowing?

S

A The wind is blowing in the direction 71.6° east of south.

B The wind is blowing in the direction 67.5° east of south.

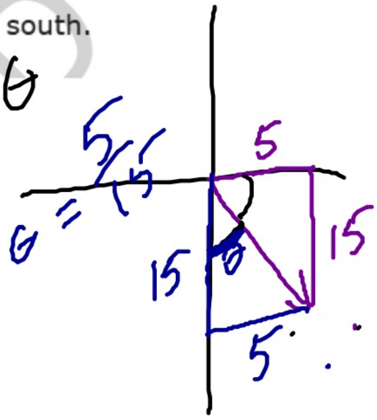
71.6°

C The wind is blowing in the direction 22.5° east of south.

D The wind is blowing in the direction 18.4° east of south.

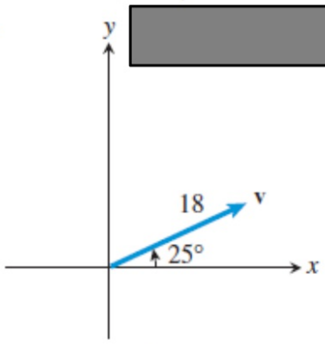
$5 = \sqrt{250} \cos \theta$

$\tan \theta = \frac{5}{15}$

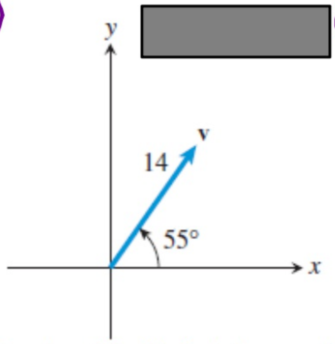


In Exercises 29–32, find the component form of the vector **v**.

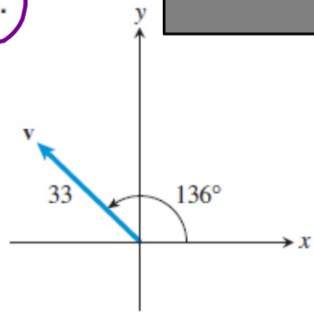
29. [redacted]



30. [redacted]



32. [redacted]



#511
30–36 even

In Exercises 33–38, find the magnitude and direction angle of the vector.

33. $\langle 3, 4\rangle$ [redacted]

34. $\langle -1, 2\rangle$ [redacted]

35. $3\mathbf{i} - 4\mathbf{j}$ [redacted]

36. $-3\mathbf{i} - 5\mathbf{j}$ [redacted]

#519

In Exercises 1–8, find the dot product of **u** and **v**.

1. $\mathbf{u} = \langle 5, 3\rangle$, $\mathbf{v} = \langle 12, 4\rangle$ [redacted]

2. $\mathbf{u} = \langle -5, 2\rangle$, $\mathbf{v} = \langle 8, 13\rangle$ [redacted]

3. $\mathbf{u} = \langle 4, 5\rangle$, $\mathbf{v} = \langle -3, -7\rangle$ [redacted]

4. $\mathbf{u} = \langle -2, 7\rangle$, $\mathbf{v} = \langle -5, -8\rangle$ [redacted]

5. $\mathbf{u} = -4\mathbf{i} - 9\mathbf{j}$, $\mathbf{v} = -3\mathbf{i} - 2\mathbf{j}$ [redacted]

6. $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j}$, $\mathbf{v} = -8\mathbf{i} + 7\mathbf{j}$ [redacted]

7. $\mathbf{u} = 7\mathbf{i}$, $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$ [redacted]

8. $\mathbf{u} = 4\mathbf{i} - 11\mathbf{j}$, $\mathbf{v} = -3\mathbf{j}$ [redacted]

In Exercises 13–22, find the angle θ between the vectors.

13. $\mathbf{u} = \langle -4, -3\rangle$, $\mathbf{v} = \langle -1, 5\rangle$ [redacted]

14. $\mathbf{u} = \langle 2, -2\rangle$, $\mathbf{v} = \langle -3, -3\rangle$ [redacted]

15. $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -3, 5\rangle$ $\approx 64.65°$

16. $\mathbf{u} = \langle 5, 2\rangle$, $\mathbf{v} = \langle -6, -1\rangle$ [redacted]

17. $\mathbf{u} = 3\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 2\sqrt{3}\mathbf{j}$ 165°

18. $\mathbf{u} = -2\mathbf{i}$, $\mathbf{v} = 5\mathbf{j}$ [redacted]

22