Warm-up

For 1-4 find the:

a) common difference/common ratio

b) explicit formula

d) 16th term

1) 66, 47, 28, 9, ...

$d = -19$ $a_n = 66 + (n-1)(-19)$ -219

2) 12, 6, 3, 1.5, ...

$r = 1/2$ $a_1 = 12$ $a_n = 12\left(\frac{1}{2}\right)^{n-1}$ $.000366$

3) 235, 345, 455, 565, ...

$d = 110$ $a_n = 235 + (n-1)110$ 1885

4) -1/3, 1, -3, 9, -27 ...

$r = -3$ $a_n = -\frac{1}{3}\cdot(-3)^{n-1}$ $a_{16} = 4,782,96$

25. **(a)** $r = 3$

(b) $a_8 = 2 \cdot 3^7 = 4374$

(c) Recursive rule: $a_1 = 2$; $a_n = 3a_{n-1}$ for $n \geq 2$

(d) Explicit rule: $a_n = 2 \cdot 3^{n-1}$

26. **(a)** $r = 2$

(b) $a_8 = 3 \cdot 2^7 = 384$

(c) Recursive rule: $a_1 = 3$; $a_n = 2a_{n-1}$ for $n \geq 2$

(d) Explicit rule: $a_n = 3 \cdot 2^{n-1}$

27. **(a)** $r = -2$

(b) $a_8 = (-2)^7 = -128$

(c) Recursive rule: $a_1 = 1$; $a_n = -2a_{n-1}$ for $n \geq$

(d) Explicit rule: $a_n = (-2)^{n-1}$

28. **(a)** $r = -1$

(b) $a_8 = -2 \cdot (-1)^7 = 2$

(c) Recursive rule: $a_1 = -2$; $a_n = -1a_{n-1} = -a_{n-1}$ for $n \geq 2$

31. $a_2 = 3 = a_1 \cdot r^1$ and $a_8 = 192 = a_1 \cdot r^7$, so $a_8/a_2 = 64 = r^6$. Therefore $r = \pm 2$, so $a_1 = 3/(\pm 2)$ $= \pm\frac{3}{2}$ and $a_n = -\frac{3}{2}\cdot(-2)^{n-1} = 3\cdot(-2)^{n-2}$ or $a_n = \frac{3}{2}\cdot 2^{n-1} = 3 \cdot 2^{n-2}$.

32. $a_3 = -75 = a_1 \cdot r^2$ and $a_6 = -9375 = a_1 \cdot r^5$, so $a_6/a_3 = 125 = r^3$. Therefore $r = 5$, so $a_1 = -75/5^2 = -3$ and $a_n = -3 \cdot 5^{n-1}$.

5. $\sum_{k=0}^{\infty} 6(-2)^k$

6. $\sum_{k=0}^{\infty} 5(-3)^k$

$\sum_{n=1}^{\infty} 5(-3)^{n-1}$

$a_1 \cdot r^{n-1}$

14. $\frac{5(1-3^{10})}{1-3} = 147{,}620$

15. $\frac{42[1-(1/6)^9]}{1-(1/6)} = 50.4(1-6^{-9}) \approx 50.4$

Objective: Write a recursive formula for arithmetic and geometric sequences.

1, 1, 2, 3, 5, 8

$a_1 = 1$

$a_2 = 1$

$a_3 = a_1 + a_2$

$a_4 = a_2 + a_3$

$a_n = a_{n-2} + a_{n-1}$

$n > 2$

Notation:

a_1 FIRST

a_{n-1}

a_{n-2}

EXAMPLE 2 ***Writing a Recursive Rule for an Arithmetic Sequence***

Write the indicated rule for the arithmetic sequence with $a_1 = 4$ and $d = 3$.

a. an explicit rule **b.** a recursive rule

$a_2 = a_1 + 3$

$a_2 = 7$

SOLUTION

a. From Lesson 11.2 you know that an explicit rule for the *n*th term of the arithmetic sequence is:

$a_n = a_1 + (n-1)d$ General explicit rule for a_n

$= 4 + (n-1)3$ Substitute for a_1 and d.

$= 1 + 3n$ Simplify.

4, 7, 10, 13,

$a_1 = 4$ $a_n = a_{n-1} + 3$

$n > 1$

b. To find the recursive equation, use the fact that you can obtain a_n by adding the common difference d to the previous term.

$a_n = a_{n-1} + d$ General recursive rule for a_n

$= a_{n-1} + 3$ Substitute for d.

A recursive rule for the sequence is $a_1 = 4$, $a_n = a_{n-1} + 3$. $n > 1$

[redacted]

Write a recursive rule for the following:

1) 3, 10, 17, 24, 31 ...

$a_1 = 3$

$a_n = a_{n-1} + 7; n > 1$

2) 56, 47, 38, 29 ...

$a_1 = 56$

$a_n = a_{n-1} - 9; n > 1$

$a_n = a_{n-1} - 9$

$a_1 = a_{1-1} - 9$

$= a_0 - 9$

EXAMPLE 3 *Writing a Recursive Rule for a Geometric Sequence*

Write the indicated rule for the geometric sequence with $a_1 = 3$ and $r = 0.1$.

a. an explicit rule **b.** a recursive rule

SOLUTION

a. From Lesson 11.3 you know that an explicit rule for the nth term of the geometric sequence is:

$a_n = a_1 r^{n-1}$ General explicit rule for a_n

$= 3(0.1)^{n-1}$ Substitute for a_1 and r.

$3, .3, .03, \frac{3}{1000}$

$a_n = a_{n-1} \cdot (.1)$

b. To write a recursive rule, use the fact that you can obtain a_n by multiplying the previous term by r.

$a_n = r \cdot a_{n-1}$ General recursive rule for a_n

$= (0.1)a_{n-1}$ Substitute for r.

$n > 1$

A recursive rule for the sequence is $a_1 = 3$, $a_n = (0.1)a_{n-1}$.

[redacted]

Write a recursive rule for the following:

1) 32, -16, 8, -4, 2, ...

2) 3, 9, 27, 81, ...

$n > 2$

$a_1 = 5$

$a_2 = 8$

$a_n = a_{n-2} \cdot a_{n-1}$ $\times 2$

$a_3 = a_{3-2} \cdot a_{3-1}$ $\times 2$

$a_3 = a_1 \cdot a_2$ $\times 2$

$a_3 = 5 \cdot 8$ $\times 2$

$a_3 = 40 \cdot a_2$ $\times 2$

$a_4 = a_3$

27) $a_{18} = 3362$ and $a_{38} = 7362$

$a_n = a_{n-1} + 200$

$a_1 = -38$

28) $a_{18} = 44.3$ and $a_{33} = 84.8$

$a_n = a_{n-1} + 2.7$

$a_1 = -1.6$

$44.3 = a_1 + (18-1)\,2.7$

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

$25 = a_1 \cdot (-5)^3$

21) $a_4 = 25,\ r = -5$

First Five Terms: $-0.2,\ 1,\ -5,\ 25,\ -125$

Explicit: $a_n = -0.2 \cdot (-5)^{n-1}$

Recursive: $a_n = a_{n-1} \cdot -5$

$a_1 = -0.2$

22) $a_1 = 4,\ r = 5$

First Five Terms: $4,\ 20,\ 100,\ 500,\ 2500$

Explicit: $a_n = 4 \cdot 5^{n-1}$

Recursive: $a_n = a_{n-1} \cdot 5$

$a_1 = 4$

EXAMPLE 4 *Writing a Recursive Rule*

Write a recursive rule for the sequence 1, 2, 2, 4, 8, 32, ...

SOLUTION

Beginning with the third term in the sequence, each term is the product of the two previous terms. Therefore, a recursive rule is given by:

$$a_1 = 1,\ a_2 = 2,\ a_n = a_{n-2} \cdot a_{n-1}$$

$n > 2$

5

[redacted]

WRITING RULES Write a recursive rule for the sequence. The sequence may be arithmetic, geometric, or neither.

35. 1, 7, 13, 19, . . .

36. 66, 33, 16.5, 8.25, . . .

37. 41, 32, 23, 14, . . .

38. 3, 8, 63, 3968, . . .

39. 33, 11, $\frac{11}{3}$, $\frac{11}{9}$, . . .

40. 7.2, 3.2, −0.8, −4.8, . . .

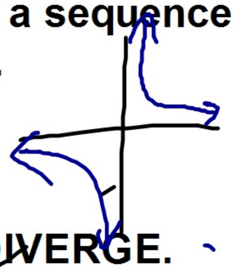
41. 2, 5, 10, 50, 500, . . .

42. 6, $6\sqrt{2}$, 12, $12\sqrt{2}$, . . .

43. 48, 4.8, 0.48, 0.048, . . .

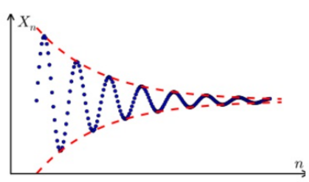
Objective: Determine characteristics of infinite sequences.

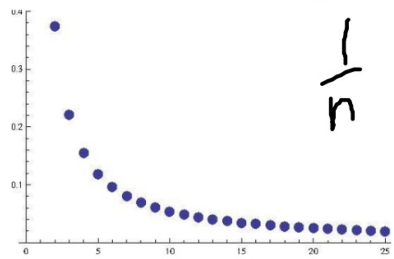
We are able to find the end behavior of a sequence, just like the end behavior of a function.



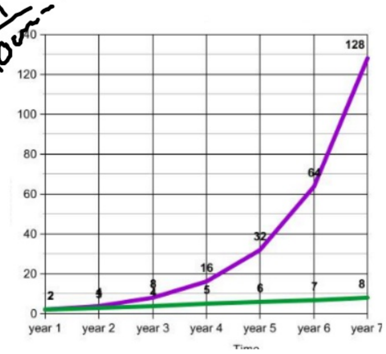
$\lim\limits_{x \to \infty}$

...CONVERGE. **OR** **...DIVERGE.**





$\frac{1}{n}$ $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{1000}$



We can find out whether a sequence converges or diverges by looking at the explicit formula.

DEFINITION Limit of a Sequence

Let $\{a_n\}$ be a sequence of real numbers, and consider $\lim_{n\to\infty} a_n$. If the limit is a finite number L, the sequence **converges** and L is the **limit of the sequence**. If the limit is infinite or nonexistent, the sequence **diverges**.

$\frac{n^3}{n^2}$

Rules for horizontal asymptotes

Bigger degree on top: diverges

Bigger degree on bottom: $\frac{n}{n^2} = \frac{1}{n}$ CONVERGES TO 0

Same degree: $\frac{3n}{n}$ CONVERGES TO 3

Use the horizontal asymptote rules to determine if the following sequences converge or diverge. If it converges, give the limit.

Determine whether the sequence converges or diverges. If it converges, give the limit.

(a) $\left\{\frac{3n}{n+1}\right\}$ CONVERGE 3 **(b)** $\left\{\frac{5n^2}{n^3+1}\right\}$ CONVERGES → 0 **(c)** $\left\{\frac{n^3+2}{n^2+n}\right\}$ DIVERGE

1	2	3	4
3/2	2	9/4	

SOLUTION

(a) Since the degree of the numerator is the same as the degree of the denominator, the limit is the ratio of the leading coefficients.

Thus $\lim_{n\to\infty} \frac{3n}{n+1} = \frac{3}{1} = 3$. The sequence converges to a limit of 3.

(b) Since the degree of the numerator is less than the degree of the denominator, the limit is zero. Thus $\lim_{n\to\infty} \frac{5n^2}{n^3+1} = 0$. The sequence converges to 0.

(c) Since the degree of the numerator is greater than the degree of the denominator, the limit is infinite. Thus $\lim_{n\to\infty} \frac{n^3+2}{n^2+n}$ is infinite. The sequence diverges.

Determine whether the sequence converges or diverges. If it converges, give the limit.

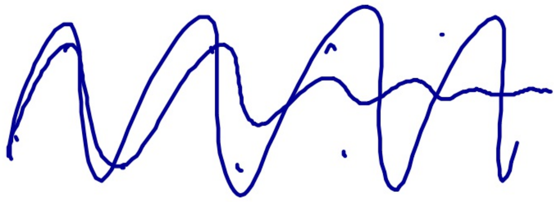
(a) $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$ CONVERGE $\to 0$

(b) $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$ $\frac{n+1}{n}$ CONVERGE TO 1

(c) 2, 4, 6, 8, 10, . . . $2n$ DIVERGE

1 2 3 4 5

(d) $-1, 1, -1, 1, \ldots, (-1)^n, \ldots$



SOLUTION

(a) $\lim_{n\to\infty} \frac{1}{n} = 0$, so the sequence converges to a limit of 0.

(b) Although the nth term is not explicitly given, we can see that $a_n = \frac{n+1}{n}$.

$\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right) = 1 + 0 = 1$. The sequence converges to a limit of 1.

(c) This time we see that $a_n = 2n$. Since $\lim_{n\to\infty} 2n = \infty$, the sequence diverges.

(d) This sequence oscillates forever between two values and hence has no limit. The sequence diverges.

Practice problems

Determine whether the sequence converges or diverges. If it converges, give the limit.

1. $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

2. $\left\{\frac{3n-1}{2-3n}\right\}$

3. $\{(0.5)^n\}$

4. $a_1 = 1$ and $a_{n+1} = a_n + 3$ for $n \geq 1$

DEFINITION Infinite Series

An **infinite series** is an expression of the form

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots.$$

THEOREM Sum of an Infinite Geometric Series

The geometric series $\sum_{k=1}^{\infty} a \cdot r^{k-1}$ converges if and only if $|r| < 1$. If it does converge, the sum is $a/(1 - r)$.

Determine whether the series converges. If it converges, give the sum.

(a) $\displaystyle\sum_{k=1}^{\infty} 3(0.75)^{k-1}$ **(b)** $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{4}{5}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{2}\right)^n$ **(d)** $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots$

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RECURSIVE