Warm-up:

Find the equation of an ellipse with:

1) Foci (2, 0) and (-2, 0) and Major axis length of 10
2) Vertices at (1, -4) & (1, 8) and Minor axis length of 8

Find the Vertices and Foci of the following graph:

3) $3x^2 + 5y^2 - 12x + 30y + 42 = 0$

$3x^2 - 12x \quad 5y^2 + 30y = -42 + 12 + 45$

$3(x^2 - 4x + 4) \quad 5(y^2 + 6y + 9)$

4) What is the unit vector for <4, -5>

$\left\langle \frac{4}{\sqrt{41}}, \frac{-5}{\sqrt{41}} \right\rangle$

Given that u = <2, -3> and v = <-1, 7>:

5) What is u • v

$-2 + -21 = \boxed{-23}$

6) What is the magnitude of 2u - v

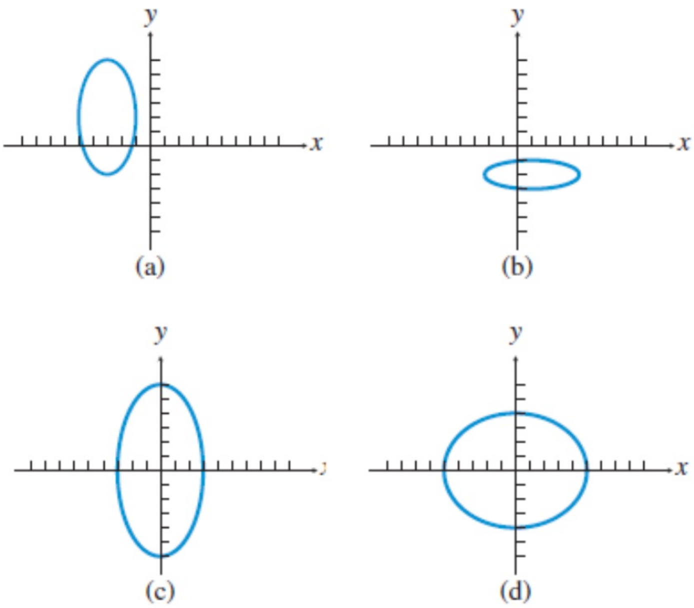
$4, -6 \quad - \langle -1, 7 \rangle$

$\langle 5, -13 \rangle$

$\sqrt{194}$

Homework Section 8.2

In Exercises 7–10, match the graph with its equation, given that the ticks on all axes are 1 unit apart.



7. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (d)

8. $\frac{y^2}{36} + \frac{x^2}{9} = 1$ (c)

9. $\frac{(y-2)^2}{16} + \frac{(x+3)^2}{4} = 1$ (a)

10. $\frac{(x-1)^2}{11} + (y+2)^2 = 1$ (b)

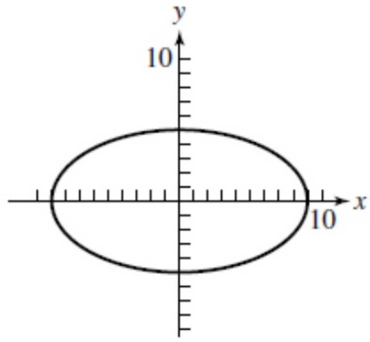
In Exercises 11–16, sketch the graph of the ellipse by hand.

12. $\dfrac{x^2}{81} + \dfrac{y^2}{25} = 1$

14. $\dfrac{y^2}{49} + \dfrac{x^2}{25} = 1$

16. $\dfrac{(x-1)^2}{2} + \dfrac{(y+3)^2}{4} = 1$

12.



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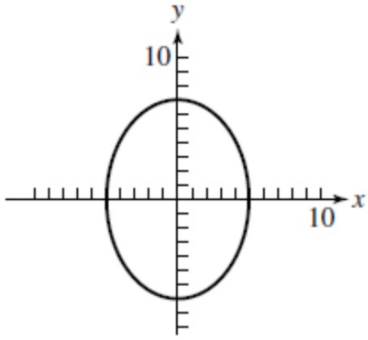

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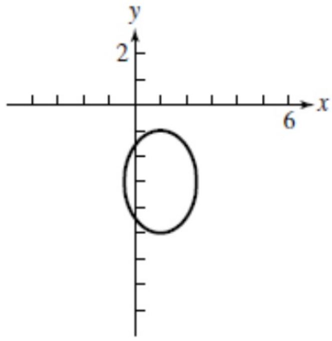


In Exercises 21–36, find an equation in standard form for the ellipse that satisfies the given conditions.

21. Major axis length 6 on y-axis, minor axis length 4

24. Foci $(0, \pm 3)$, major axis length 10 $y^2/25 + x^2/16 = 1$

27. Major axis endpoints $(0, \pm 6)$, minor axis length 8

30. Minor axis endpoints $(\pm 12, 0)$, major axis length 26

21. $\dfrac{y^2}{9} + \dfrac{x^2}{4} = 1$

27. $\dfrac{y^2}{36} + \dfrac{x^2}{16} = 1$

30. $\dfrac{y^2}{169} + \dfrac{x^2}{144} = 1$

33. The foci are $(1, -4)$ and $(5, -4)$; the major axis end $(0, -4)$ and $(6, -4)$. $(x-3)^2/9 + (y+4)^2/5 = 1$

36. The major axis endpoints are $(-5, 2)$ and $(3, 2)$; the minor axis length is 6. $(x+1)^2/16 + (y-2)^2/9 = 1$

In Exercises 37–40, find the center, vertices, and foci of the ellipse.

38. $\dfrac{(x-3)^2}{11} + \dfrac{(y-5)^2}{7} = 1$

38. Center: $(3, 5)$; Vertices: $(3 \pm \sqrt{11}, 5)$; Foci: $(1, 5)$, $(5, 5)$

40. $\dfrac{(y-1)^2}{25} + \dfrac{(x+2)^2}{16} = 1$

40. Center: $(-2, 1)$; Vertices: $(-2, 6)$, $(-2, -4)$; Foci: $(-2, 4)$, $(-2, -2)$

In Exercises 45–48, prove that the graph of the equation is an ellipse, and find its vertices, foci, and eccentricity.

46. $3x^2 + 5y^2 - 12x + 30y + 42 = 0$

48. $4x^2 + y^2 - 32x + 16y + 124 = 0$

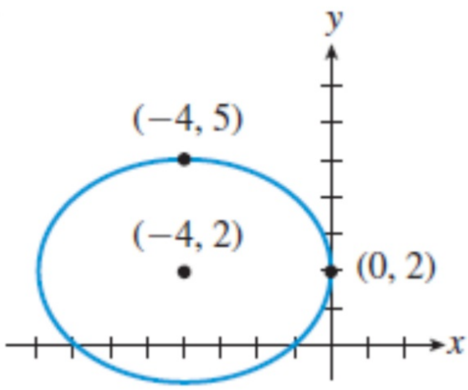
46. Vertices: $(2 \pm \sqrt{5}, -3)$; Foci: $(2 \pm \sqrt{2}, -3)$;

48. Vertices: $(4, -10)$, $(4, -6)$;

Foci: $(4, -8 \pm \sqrt{3})$;

In Exercises 49 and 50, write an equation for the ellipse.

50.



50. $\dfrac{(x+4)^2}{16} + \dfrac{(y-2)^2}{9} = 1$

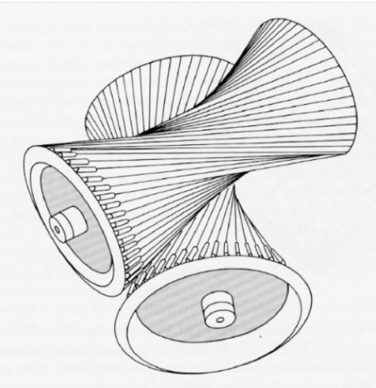
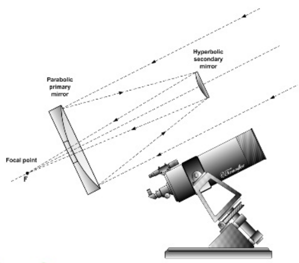
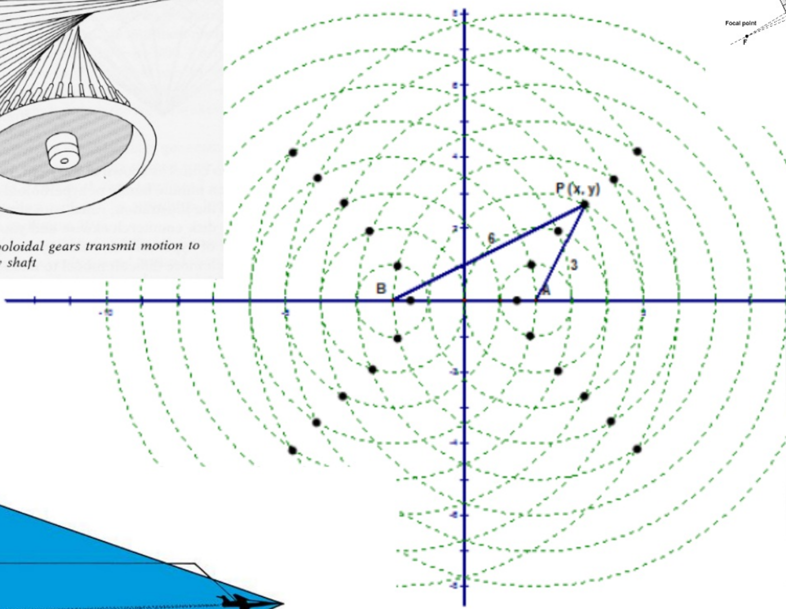
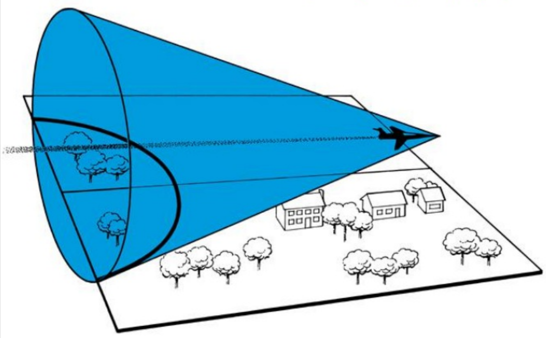


Figure 102 Hyperboloidal gears transmit motion to a skew shaft







8.3 Hyperbolas

Objective: To write equations for and graph hyperbolas

A hyperbola is the set of all points in a plane such that the absolute value of the difference of the distances from two fixed points, called the foci, is constant.

Center: (h, k)

MAJOR

Transverse Axis: Segment with length 2a that stretches from the vertices of the hyperbola

MINOR

Conjugate Axis: Segment with length 2b that is perpendicular to the transverse axis

Standard form of a Hyperbola:

$$\frac{(x-h)^2}{a^2} \boxed{-} \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} \boxed{-} \frac{(x-h)^2}{b^2} = 1$$

	Horizontal hyperbola	**Vertical hyperbola**
Standard form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Equation of the asymptotes	$y-k = \pm \frac{b}{a}(x-h)$	$y-k = \pm \frac{a}{b}(x-h)$

How do you determine "a" and "b?"

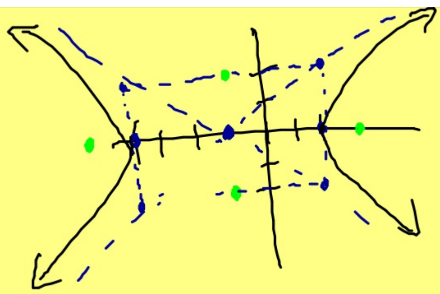
a^2 is always first!!!
NOT ALWAYS BIGGER THAN B!

How do you determine if the hyperbola is horizontal or vertical?

Horizontal if x is first

Vertical if y is first

How do you graph an hyperbola?



STEPS
1. Graph the center.
2. From the center, move left/right or up/down "a" units (these 2 points are the vertices & the endpoints of the transverse axis).
3. From the vertices, move up/down or left/right "b" units (these 4 points form a rectangle; the diagonals are the asymptotes).
4. Draw the hyperbola thru the vertices approaching the asymptotes.

GUIDED PRACTICE
Find the center, the vertices, and the equation of the asymptotes. Then graph.
1)

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

$a^2 + b^2 = c^2$

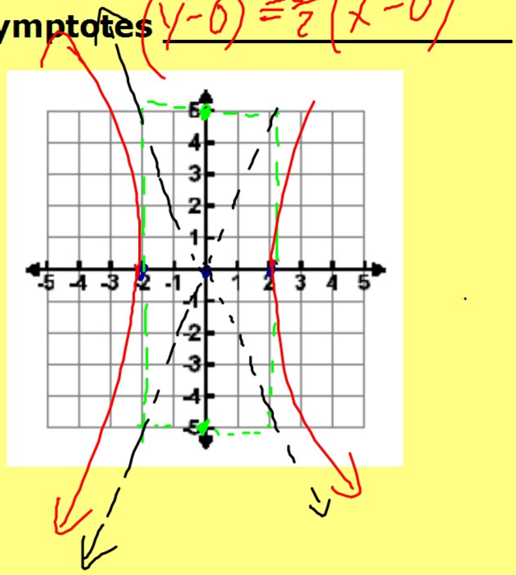
$\sqrt{29}$

F: $(\pm\sqrt{29}, 0)$

center $(0,0)$

vertices $(\pm 2, 0)$

asymptotes $(y-0) = \pm\frac{5}{2}(x-0)$



3. Finding the Vertices and Foci of a Hyperbola

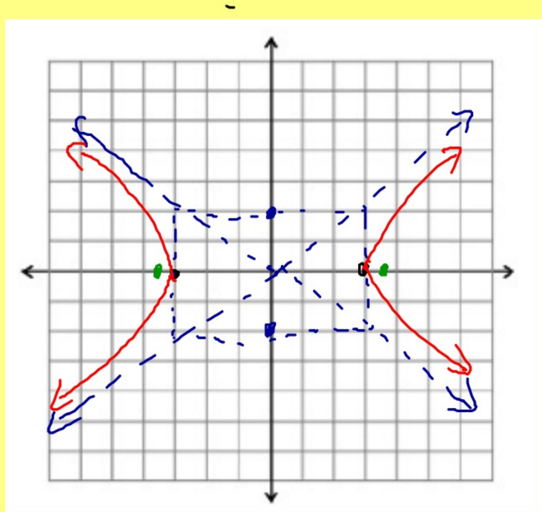
$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Find the vertices and the foci of the hyperbola $\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$.

V: $(\pm 3, 0)$

F: $(\pm\sqrt{13}, 0)$

$y = \pm\frac{2}{3}x$



19–20 ■ Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph.

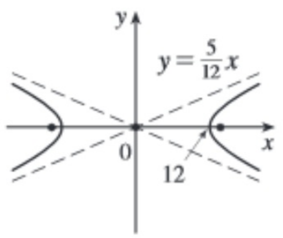
19. $\dfrac{x^2}{144} - \dfrac{y^2}{25} = 1$

20. $\dfrac{y^2}{16} - \dfrac{x^2}{36} = 1$

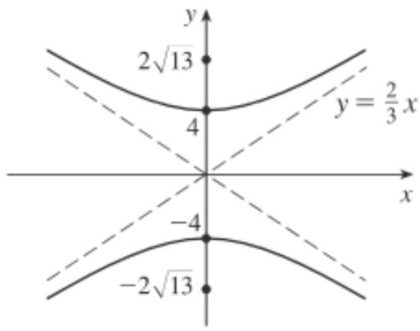
21. $y^2 - x^2 = 4$

~~**22.**~~ $\cancel{9x^2 - 4y^2 = 36}$

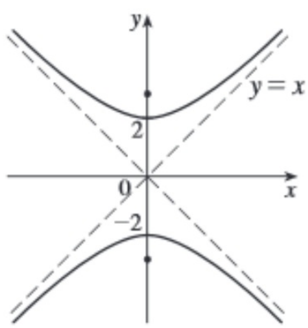
19. $(\pm 12, 0)$, $(\pm 13, 0)$, $y = \pm \frac{5}{12} x$



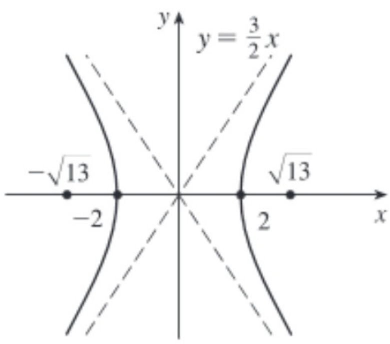
20. $(0, \pm 4)$, $(0, \pm 2\sqrt{13})$, $y = \pm \frac{2}{3} x$



21. $(0, \pm 2)$, $(0, \pm 2\sqrt{2})$, $y = \pm x$



22. $(\pm 2, 0)$, $(\pm\sqrt{13}, 0)$, $y = \pm \frac{3}{2} x$



8.

Locating Key Points of a Hyperbola

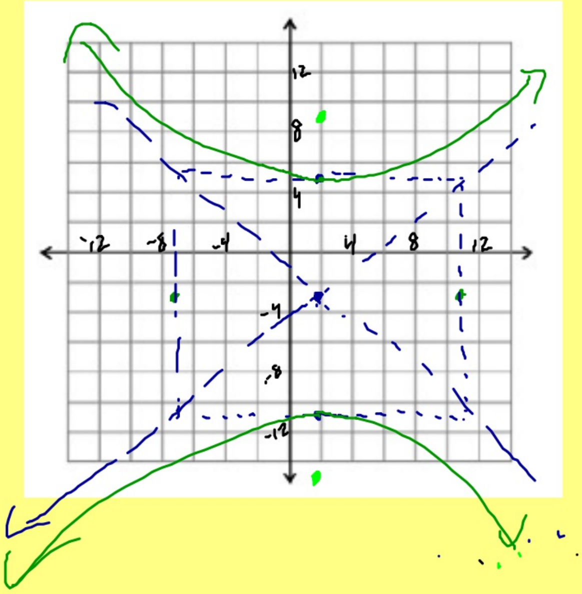
find the center, vertices, and the foci of the hyperbola.

$$\frac{(y+3)^2}{64} - \frac{(x-2)^2}{81} = 1$$

$(2,-3)$

V) $(2,5)$ $(2,-11)$

F) $(2,-3 \pm \sqrt{145})$



4.

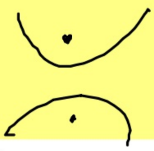
Finding an Equation and Graphing a Hyperbola

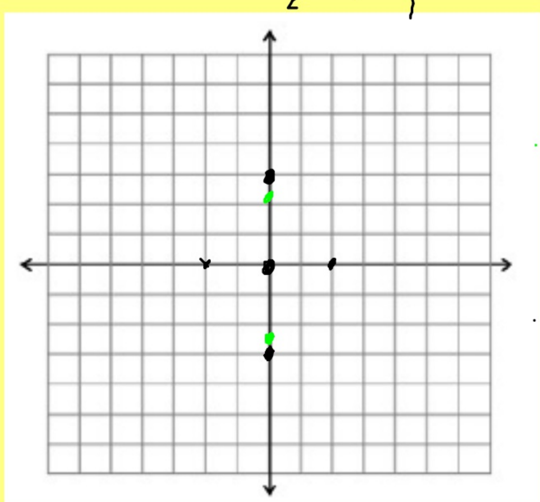
Find an equation of the hyperbola with foci $(0, -3)$ and $(0, 3)$ whose conjugate axis has length 4. ~~Sketch the hyperbola and~~ its asymptotes, ~~and support your sketch with a grapher.~~

$$\frac{y^2}{5} - \frac{x^2}{4} = 1$$

$a^2 + b^2 = c^2$

$a^2 + 4 = 9$

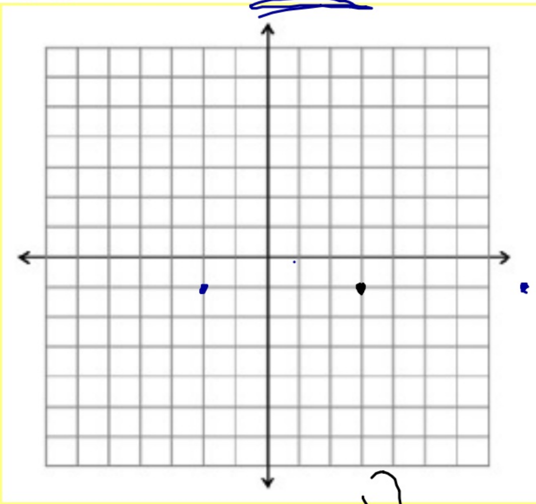




6. Finding an Equation of a Hyperbola

Find the standard form of the equation for the hyperbola whose transverse axis has endpoints $(-2, -1)$ and $(8, -1)$, and whose conjugate axis has length 8.

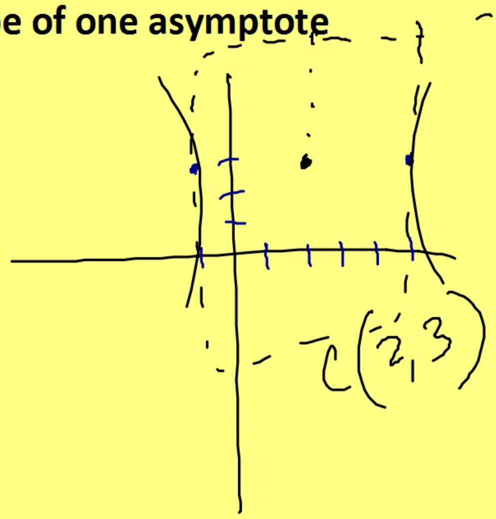
$$\frac{(x-3)^2}{25} - \frac{(y+1)^2}{16} = 1$$



$(3, -1)$

7. Find the equation of the hyperbola whose transverse axis endpoints (-1,3) and (5,3) , slope of one asymptote is 4/3.

$$\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$$



43. Hyperbola, foci $(0, \pm 3)$, vertices $(0, \pm 1)$

44. Hyperbola, foci $(\pm 6, 0)$, vertices $(\pm 4, 0)$

45. Hyperbola, foci $(1, 3)$ and $(7, 3)$, vertices $(2, 3)$ and $(6, 3)$

46. Hyperbola, foci $(2, -2)$ and $(2, 8)$, vertices $(2, 0)$ and $(2, 6)$

47. Hyperbola, vertices $(\pm 3, 0)$, asymptotes $y = \pm 2x$

43. $y^2 - \frac{1}{8}x^2 = 1$ **44.** $\frac{1}{16}x^2 - \frac{1}{20}y^2 = 1$

45. $\dfrac{(x-4)^2}{4} - \dfrac{(y-3)^2}{5} = 1$

46. $\frac{1}{9}(y-3)^2 - \frac{1}{16}(x-2)^2 = 1$ **47.** $\frac{1}{9}x^2 - \frac{1}{36}y^2 = 1$

11.

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation

$$-y^2 + x^2 + 6x + 10y - 17 = 0$$

$9x^2 - y^2 - 36x - 6y + 18 = 0$

$9y^2 - x^2 + 2x + 54y = -62$