Warm up [redacted]

Given two terms in a geometric sequence find the 8th term and the recursive formula.

1. $a_4 = -12$ and $a_5 = -6$

$-12 = a_1 \cdot \left(\frac{1}{2}\right)^3$

$a_8 = -\frac{3}{4}$

Recursive: $a_n = a_{n-1} \cdot \boxed{\frac{1}{2}}$

$a_1 = -96$

$a_8 = -96 \cdot \frac{1}{2}^7$

2. $a_1 = -2$ and $a_5 = -512$

$a_8 = 32768$

Recursive: $a_n = a_{n-1} \cdot -4$

$a_1 = -2$

$-512 = -2 \cdot r^4$

Given a term in an arithmetic sequence and the common difference find the recursive formula and the three terms in the sequence after the last one given.

a $\quad -1.4 = a_1 + (20) \cdot 6$

3. $a_{\underline{21}} = -1.4,\ d = 0.6$

Next 3 terms: –0.8, –0.2, 0.4

Recursive: $a_n = a_{n-1} + 0.6$

$a_1 = -13.4$

4. $a_{22} = -44,\ d = -2$

Next 3 terms: –46, –48, –50

Recursive: $a_n = a_{n-1} - 2$

$a_1 = -2$

[redacted]

Homework Answers

) $d = 4$

) $a_{10} = 6 + 9(4) = 42$

) Recursive rule: $a_1 = 6$; $a_n = a_{n-1} + 4$ for $n \geq 2$

) Explicit rule: $a_n = 6 + 4(n - 1)$

) $d = 5$

) $a_{10} = -4 + 9(5) = 41$

) Recursive rule: $a_1 = -4$; $a_n = a_{n-1} + 5$ for $n \geq 2$

) Explicit rule: $a_n = -4 + 5(n - 1)$

) $d = 3$

) $a_{10} = -5 + 9(3) = 22$

) Recursive rule: $a_1 = -5$; $a_n = a_{n-1} + 3$ for $n \geq 2$

) Explicit rule: $a_n = -5 + 3(n - 1)$

) $d = 11$

) $a_{10} = -7 + 9(11) = 92$

) Recursive rule: $a_1 = -7$; $a_n = a_{n-1} + 11$ for $n \geq 2$

) Explicit rule: $a_n = -7 + 11(n - 1)$

25. **(a)** $r = 3$

(b) $a_8 = 2 \cdot 3^7 = 4374$

(c) Recursive rule: $a_1 = 2$; $a_n = 3a_{n-1}$ for $n \geq 2$

(d) Explicit rule: $a_n = 2 \cdot 3^{n-1}$

26. **(a)** $r = 2$

(b) $a_8 = 3 \cdot 2^7 = 384$

(c) Recursive rule: $a_1 = 3$; $a_n = 2a_{n-1}$ for $n \geq 2$

(d) Explicit rule: $a_n = 3 \cdot 2^{n-1}$

27. **(a)** $r = -2$

(b) $a_8 = (-2)^7 = -128$

(c) Recursive rule: $a_1 = 1$; $a_n = -2a_{n-1}$ for $n \geq 2$

(d) Explicit rule: $a_n = (-2)^{n-1}$

28. **(a)** $r = -1$

(b) $a_8 = -2 \cdot (-1)^7 = 2$

(c) Recursive rule: $a_1 = -2$; $a_n = -1a_{n-1} = -a_{n-1}$

Practice problems

Determine whether the sequence converges or diverges. If it converges, give the limit.

1. $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$ Converges to 0 $\quad \frac{1}{n^2}$

   1 2 3 4

2. $\left\{\frac{3n-1}{2-3n}\right\}$ Converges to -1

3. $\{(0.5)^n\}$ Converges to 0 $\quad \frac{1}{2} \; \frac{1}{4} \; \frac{1}{8}$

$\frac{n^2}{n^3}$ CONVERGES TO 0

$\frac{2n^2}{n^2}$ = CONVERGE TO 2

$\frac{n^3}{n^2} \Rightarrow$ NOT DIVERGE

DEFINITION Infinite Series

An **infinite series** is an expression of the form

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots.$$

3 1.5 .75

. 0 1

$1+2+3+4+5 \ldots = -\frac{1}{12}$

THEOREM Sum of an Infinite Geometric Series

The geometric series $\sum_{k=1}^{\infty} a \cdot r^{k-1}$ converges if and only if $|r| < 1$. If it does converge, the sum is $a/(1-r)$.

$S_n = \frac{a}{1-r}$

$5(.5)^n$

$5 \cdot (1.5)^n$

Determine whether the series converges. If it converges, give the sum.

(a) $\sum_{k=1}^{\infty} 3(0.75)^{k-1}$ (b) $\sum_{n=0}^{\infty} \left(-\frac{4}{5}\right)^n$ C

(c) $\sum_{n=1}^{\infty} \left(\frac{\pi}{2}\right)^n$ D (d) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

2

$\frac{a_1}{1-r}$ $\frac{3}{1-.75}$

C $\frac{1}{9/5} = \frac{1}{1-\frac{-4}{5}}$

$\frac{1}{1-.5}$

$a_1 \cdot r^{n-1}$ 5/9

SOLUTION

(a) Since $|r| = |0.75| < 1$, the series converges. The first term is $3(0.75)^0 = 3$, so the sum is $a/(1-r) = 3/(1-0.75) = 12$.

(b) Since $|r| = |-4/5| < 1$, the series converges. The first term is $(-4/5)^0 = 1$, so the sum is $a/(1-r) = 1/(1-(-4/5)) = 5/9$.

(c) Since $|r| = |\pi/2| > 1$, the series diverges.

(d) Since $|r| = |1/2| < 1$, the series converges. The first term is 1, and so the sum is $a/(1-r) = 1/(1-1/2) = 2$.

Sequence and series with factorials

$a_n = n!$

$8!$

$8 \cdot 7 \cdot 6 \cdot 5$

Simplify factorials

$\frac{3!}{6!}$ $\frac{12!}{6!8!}$ $\frac{(n+1)!}{n!} = n+1$

$\frac{n+1 \cdot \cancel{n \cdot n-1 \cdot n-2}}{\cancel{n \cdot n-1 \cdot n-2 \cdot n-3}}$

Example 1: Simplify $\frac{n!}{(n-2)!}$

Example 2: Simplify $\frac{(n+1)!}{(n+3)!}$

$$\frac{n!}{(n-2)!} = n^2 - n$$

$$\frac{n(n-1)\cdot \cancel{n-2}\cdot \cancel{n-3}}{\cancel{n-2}\cdot \cancel{n-3}}$$

$$\frac{(n+1)!}{(n+3)!} = \frac{1}{n^2+5n+6}$$

Example 3: Simplify $\frac{(k+2)!}{(k-1)!}$

$$\frac{(k+2)!}{(k-1)!} = k^3 + 3k^2 + 2k$$