

## Warm up

1. To approximate the length of a marsh, a surveyor walks 250m from point A to point B, then turns and walks 220m to point C. If angle B is  $105^\circ$ , find the length of AC.

$$(2350)^2 = (2323)^2$$

2. The distance on a map from the airport in Miami, FL to the one in Nassau, Bahamas is 295 kilometers due east. Bangor, Maine is northeast of both cities; its airport is 2350 kilometers from Miami and 2323 kilometers from Nassau. What bearing would a plane need to take to fly from Nassau to Bangor?

$$91.5^\circ$$

## Homework:

7) 4585.8 feet

8) 171.5 meters

3) 5.18 meters

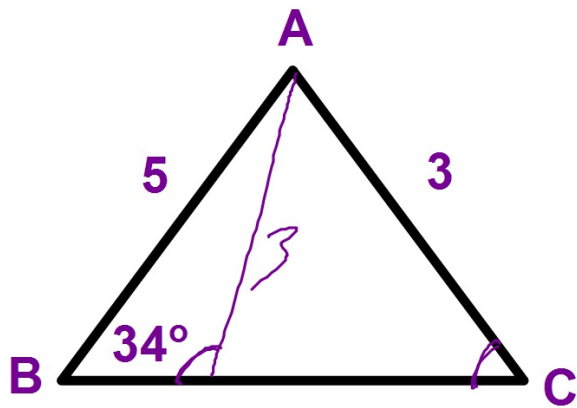
4) 120 meters; 105.5 meters

5) 157.7 meters

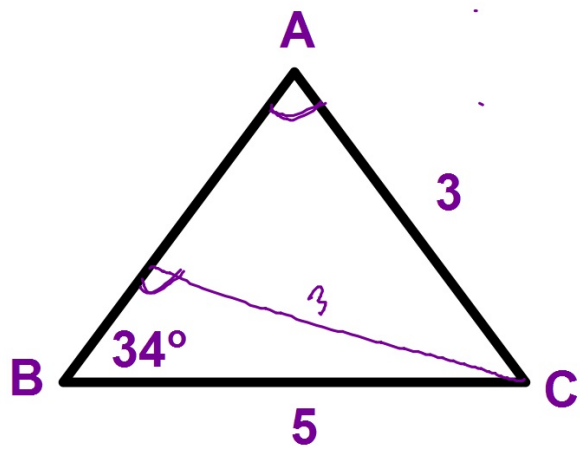
7) 8.4 km

Correction...

1)  $B = 34^\circ$   $b = 3$   $c = 5$



2)  $B = 34^\circ$   $b = 3$   $a = 5$



# 5.1 Fundamental Identities

Objective: Simplify expressions using trigonometric identities.

## Fundamental Identities

### Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \qquad \cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \qquad \tan \theta = \frac{1}{\cot \theta}$$

### Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

## Pythagorean Identities



$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

## CoFunction Identities

$$\sin \left( \frac{\pi}{2} - \theta \right) = \cos \theta$$

$$\cos \left( \frac{\pi}{2} - \theta \right) = \sin \theta$$

$$\csc \left( \frac{\pi}{2} - \theta \right) = \sec \theta$$

$$\sec \left( \frac{\pi}{2} - \theta \right) = \csc \theta$$

$$\tan \left( \frac{\pi}{2} - \theta \right) = \cot \theta$$

$$\cot \left( \frac{\pi}{2} - \theta \right) = \tan \theta$$

## Even odd Identities

$$\sin(-\theta) = -\sin \theta$$

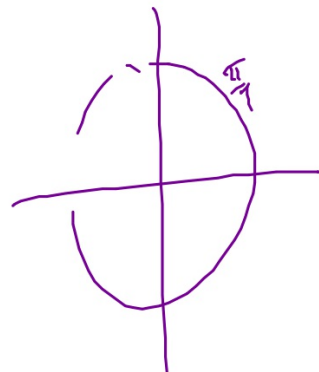
$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$



**Simplify:**  $(\sec x)(\tan^2 x) + \sec x$

$$\sec x (\tan^2 x + 1)$$

$$\sec x (\sec^2 x)$$

$$\boxed{\sec^3 x}$$



Factor each expression.

a.  $\sec^2 \theta - 1$

$$(\sec \theta + 1)(\sec \theta - 1)$$

b.  $4 \tan^2 \theta + \tan \theta - 3$

$$4u^2 + u - 3$$

$$(4u - 3)(u + 1)$$

$$(4 \tan \theta - 3)(\tan \theta + 1)$$

$$u = \tan \theta$$

Factor  $(\csc^2 x) - \cot x - 3$ .

$$(1 + \cot^2 x) - \cot x - 3$$

$$\cot^2 x - \cot x - 2$$

$$(\cot x - 2)(\cot x + 1)$$

Simplify  $\sin t + \cot t \cos t$ .

$$\sin t + \frac{\cos t}{\sin t} \cdot \frac{\cos t}{1}$$

$$\frac{\sin t}{\sin t} \cdot \frac{\sin t}{1} + \frac{\cos^2 t}{\sin t}$$

$$\frac{\sin^2 t}{\sin t} + \frac{\cos^2 t}{\sin t}$$

$$\frac{1}{\sin t} \quad \boxed{\csc t}$$

Perform the addition and simplify.

$$\frac{(1-\cos\theta) \sin\theta}{1-\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta - \cos\theta \sin\theta}{\cancel{1-\cos\theta}} + \frac{\cos\theta}{\sin\theta} \frac{\sin\theta}{\sin\theta}$$

*sin<sup>2</sup>θ*

$$\frac{\cancel{\sin\theta - \cos\theta \sin\theta} + \cancel{\cos\theta \sin\theta}}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} = \csc\theta$$

Rewrite  $\frac{1}{1 + \sin x}$  so that it is *not* in fractional form.

$$\frac{1 - \sin x}{1 - \sin^2 x}$$

$$\frac{1 - \sin x}{\cos^2 x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$

$$\sec^2 x - \tan x \cdot \sec x$$

**Simplify the following expressions.**

1)  $\sin x \cot x$

5)  $\cos x + \tan x \sin x$

2)  $\frac{\sec x}{\csc x}$

6)  $\sin^3 x + \sin x \cos^2 x$

3)  $\frac{1 - \sin^2 x}{\cos x}$

7)  $\frac{\csc x - \sin x}{\csc x}$

4)  $\sin t - \sin t \cos^2 t$

8)  $\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$

$$1) \sin x \cot x = \sin x \cdot \frac{1}{\tan x} = \cos x$$

$$2) \frac{\sec x}{\csc x} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$$

$$3) \frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$$

$$4) \sin t - \sin t \cos^2 t = \sin t \cdot (1 - \cos^2 t) = \sin t \cdot \sin^2 t = \sin^3 t$$

$$5) \cos x + \tan x \sin x = \cos x + \frac{\sin x}{\cos x} \cdot \sin x = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$$

$$6) \sin^3 x + \sin x \cos^2 x = \sin x \cdot (\sin^2 x + \cos^2 x) = \sin x$$

$$7) \frac{\csc x - \sin x}{\csc x} = \frac{\csc x}{\csc x} - \frac{\sin x}{\csc x} = 1 - \sin^2 x = \cos^2 x$$

$$8) \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{\sin x \cdot (1 + \sin x) + \cos x \cdot \cos x}{\cos x \cdot (1 + \sin x)} = \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x \cdot (1 + \sin x)} = \frac{\sin x + 1}{\cos x \cdot (1 + \sin x)} = \frac{1}{\cos x} = \sec x$$

## **Exit Ticket:**

**1. A plane flies at a bearing of N35°E for 250 km, it then changes bearings and flies directly east for 450 km. How far is the plane from where it started?**

**2. Solve the triangle for all possible solutions**

$$\mathbf{A = 32^\circ \quad a = 6 \quad c = 4}$$

**3) Simplify:**