

WARM-UP

Describe and graph the transformation for each:

1. $g(x) = 2(x - 3)^2 - 4$

2. $f(x) = -3\sqrt{x+1}$

Write an equation for the following transformations:

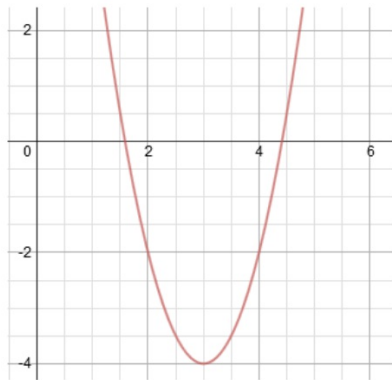
3. $y = x^2$: a shift left 3 units, then a vertical stretch by a factor of 2.

4. $y = |x|$: shift right 4 units, reflected over the x-axis, and finally a shift down 2 units.

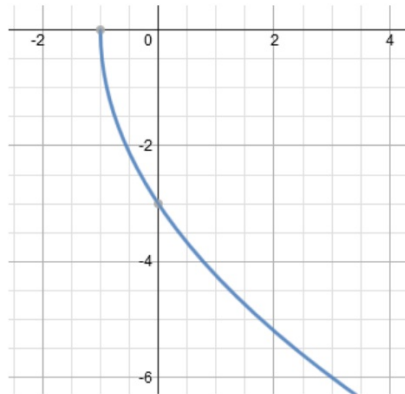
5. $f(x) = \frac{\sqrt{x+3}}{x+5}$ **Give the Domain**

WARM-UP

1)



2)



3) $2(x + 3)^2$

4) $-|x - 4| - 2$

5) $[-3, \infty)$

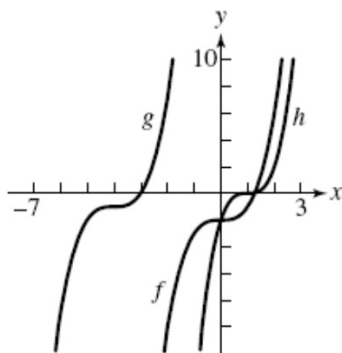
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2. Vertical translation up 5.2 units

4. Horizontal translation right 3 units

10. Horizontal translation right 5 units

22.



25. Since the graph is translated left 5 units, $f(x) = \sqrt{x+5}$.

26. The graph is reflected across the y -axis and translated right 3 units. $y = \sqrt{-x}$ would be reflected across the y -axis; the horizontal translation gives

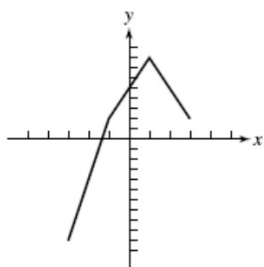
$$f(x) = \sqrt{-(x-3)} = \sqrt{3-x}.$$

See also Exercise 12 in this section, and note accompanying that solution.

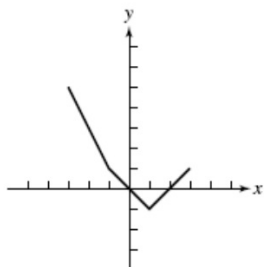
27. The graph is reflected across the x -axis, translated left 2 units, and translated up 3 units. $y = -\sqrt{x}$ would be reflected across the x -axis, $y = -\sqrt{x+2}$ adds the horizontal translation, and finally, the vertical translation gives $f(x) = -\sqrt{x+2} + 3 = 3 - \sqrt{x+2}$.

28. The graph is vertically stretched by 2, translated left 5 units, and translated down 3 units. $y = 2\sqrt{x}$ would be vertically stretched, $y = 2\sqrt{x+5}$ adds the horizontal translation, and finally, the vertical translation gives $f(x) = 2\sqrt{x+5} - 3$.

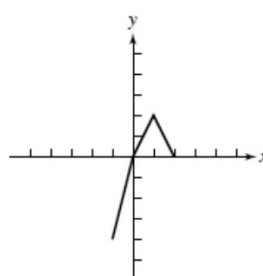
51. Translate left 1 unit, then vertically stretch by 3, and finally translate up 2 units. The four vertices are transformed to $(-3, -10)$, $(-1, 2)$, $(1, 8)$, and $(3, 2)$.



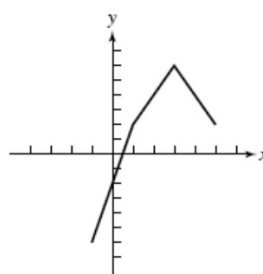
52. Translate left 1 unit, then reflect across the x -axis, and finally translate up 1 unit. The four vertices are transformed to $(-3, 5)$, $(-1, 1)$, $(1, -1)$, and $(3, 1)$.



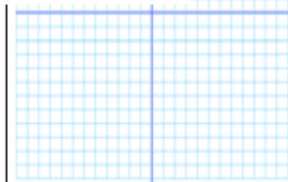
53. Horizontally shrink by $\frac{1}{2}$. The four vertices are transformed to $(-1, -4)$, $(0, 0)$, $(1, 2)$, and $(2, 0)$.



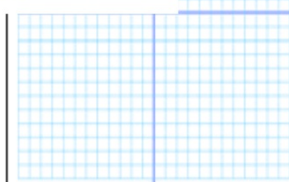
54. Translate right 1 unit, then vertically stretch by 2, and finally translate up 2 units. The four vertices are transformed to $(-1, -6)$, $(1, 2)$, $(3, 6)$, and $(5, 2)$.



$$f(x) = \begin{cases} 3, & x \leq -2 \\ -2 - 4x & -2 < x < 3 \\ x^2 - 1, & x \geq 3 \end{cases}$$



$$f(x) = \begin{cases} 3x - 2, & x \leq -2 \\ x^2 + 1, & -2 < x < 1 \\ 6, & x \geq 1 \end{cases}$$



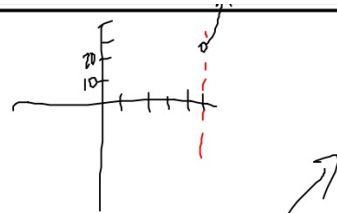
Also list **Domain**, **Range**, **Continuity** and evaluate:

$$f(-2) \quad f(0) \quad f(4)$$

for each piecewise function above

$$25 = 5c + 7$$

$$c = \frac{18}{5}$$



$$53 = 44 + c$$

$$\text{Let } g(x) = \begin{cases} 4x + c, & \text{if } x < 11 \\ 5x - 2, & \text{if } x \geq 11 \end{cases}$$



What is the value of c that will make $f(x)$ continuous at $x = 11$?

$$c = 9$$

1-4 Operations and Compositions of Functions

$$f(x) = x^2 - 4$$

$$g(x) = x + 2$$

Find **a. $f(x) + g(x)$**

$$(f+g)(x)$$
$$x^2 - 4 + x + 2$$

$$x^2 + x - 2$$

b. $f(x) - g(x)$

$$(f-g)(x)$$
$$x^2 - 4 - (x + 2)$$

$$x^2 - 4 - x - 2$$

$$x^2 - x - 6$$

c. $f(x) \cdot g(x)$

$$(f \cdot g)(x)$$
$$(x^2 - 4)(x + 2)$$

$$x^3 + 2x^2 - 4x - 8$$

d. $f(x)/g(x)$

$$\left(\frac{f}{g}\right)(x)$$
$$\frac{x^2 - 4}{x + 2}$$

$$\frac{(x+2)(x-2)}{x+2}$$

$$x - 2$$

Other Notation

What is the notation for *composition of functions* ?

$(f \circ g)(x)$ is read " **f** of **g** of **x** ".

$(f \circ g)(x)$ can also be written as **$f(g(x))$**

x is the **input**

f(x) or **y** is the **output**

$$(f \circ g)(x)$$

$$(f \cdot g) x$$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

1. $f(x) = 2x$ and $g(x) = x + 2$

x
 $f(x)$

$$f(x+2)$$

$$(f \circ g)(x)$$

$$g(2x)$$

$$\begin{aligned} f(x) &= 2x \\ f(g(x)) &= 2(x+2) \\ &= 2x+4 \end{aligned}$$

$$\begin{aligned} g(x) &= x+2 \\ g(f(x)) &= (2x)+2 \end{aligned}$$

$$\underline{f(g(x))}$$

$$g(f(x))$$

2. $f(x) = x^2 + x - 1$ and $g(x) = x + 3$ $f(g(x))$ $g(f(x))$

$$f(g(x)) = (x+3)^2 + (x+3) - 1$$

$$g(f(x)) = (x^2 + x - 1) + 3$$

$$g(f(x)) = x^2 + x + 2$$

If $f(x) = x + 2$, $g(x) = 2x - 1$, and $h(x) = x^2 + x - 3$,
find each value:

3. $g(f(0))$

$$g(2) = 3$$

4. $f(h(-4))$

$$f(9) = 11$$

5. $h(g(3))$

$$h(5) = 27$$

6. $f(g(-3))$

$$f(-7) = -5$$

For each function h , find functions f and g such that $h(x) = f(g(x))$.

(a) $h(x) = (x + 1)^2 - 3(x + 1) + 4$

$f(g(x))$

$f(x) = x^2 - 3x + 4$

$g(x) = (x+1)$

(b) $h(x) = \sqrt{x^3 + 1}$

$f(x) = \sqrt{x}$

$g(x) = x^3 + 1$

$f(g(x))$

$\sqrt{x^3 + 1}$

$f(x^3 + 1)$

$$f(x) = 2x^2 \text{ and } g(x) = 3 - x$$

Extra
Practice

Find $(f \circ g)(x)$ and $(g \circ f)(x)$

$$2(3-x)^2$$

$$3 - 2x^2$$

Find $(f \circ g)(5)$ and $(g \circ f)(2)$ -5

$$8$$

$$g(f(2))$$

Find $(g \circ g)(1)$ and $(f \circ f)(-1)$

Challenge: Find $g(f(3)) - f(g(4))$ -15 2 -17

Wrap up

1. Given $f(x) = x^3$ consider the following transformation: reflected over the x axis, shifted 3 left and 4 down.

a. Write the equation b. graph without a calculator

2. Describe the transformation of $g(x) = -2|x+3|$ from the parent function $f(x) = |x|$

3. Determine if the function $f(x) = x^5 - x^3 + 2$ is even odd or neither. Justify your answer

Find the Domain of the following functions

1. $f(x) = \frac{\sqrt{x-5}}{x+2}$

2. $f(x) = \frac{4-x}{\sqrt{x-3}}$

3. $f(x) = \frac{\sqrt{5-x}}{x+2}$

