Warm up

1. Get ready for the quiz...

After you finish...

- Turn in your quiz in 4th Block Basket up front
- Work on compositions below:

$g(a) = a^2 + 3$
$h(a) = 4a + 4$
Find $g(h(a))$

$16a^2 + 32a + 19$

[redacted]

$h(t) = -2t - 5$
$g(t) = -3t^2 - 2t$
Find $h(g(t))$

$6t^2 + 4t - 5$

[redacted]

$f(t) = -2t$
$g(t) = t^2 - 4t$
Find $(f - g)(9)$

-63 [redacted]

$h(n) = -2n + 3$
$g(n) = 4n$
Find $(h \circ g)(-2)$

19 [redacted]

In Exercises 1–4, find formulas for the functions $f+g$, $f-g$, and fg. Give the domain of each.

1. $f(x) = 2x - 1$; $g(x) = x^2$ **2.** $f(x) = (x-1)^2$; $g(x) = 3 - x$

3. $f(x) = \sqrt{x}$; $g(x) = \sin x$ **4.** $f(x) = \sqrt{x+5}$; $g(x) = |x+3|$

$f + g)(x) = 2x - 1 + x^2$; $(f - g)(x) = 2x - 1 - x^2$; $(fg)(x) = (2x - 1)(x^2) = 2x^3 - x^2$. Th
ree domains are $(-\infty, \infty)$.
$f + g)(x) = x^2 - 3x + 4$; $(f - g)(x) = x^2 - x - 2$; $(fg)(x) = (x - 1)^2(3 - x) = -x^3 + 5x^2$
ains, so all three domains are $(-\infty, \infty)$.
$f + g)(x) = \sqrt{x} + \sin x$; $(f - g)(x) = \sqrt{x} - \sin x$; $(fg)(x) = \sqrt{x}\sin x$. Domain in each case
$f + g)(x) = \sqrt{x+5} + |x + 3|$; $(f - g)(x) = \sqrt{x+5} - |x + 3|$; $(fg)(x) = \sqrt{x+5}\,|x + 3|$.

In Exercises 5–8, find formulas for f/g and g/f. Give the domain of each.

5. $f(x) = \sqrt{x + 3}$; $g(x) = x^2$

6. $f(x) = \sqrt{x - 2}$; $g(x) = \sqrt{x + 4}$

$) = \dfrac{\sqrt{x+3}}{x^2}$; $x + 3 \geq 0$ and $x \neq 0$, so the domain is $[-3, 0) \cup (0, \infty)$.

$) = \dfrac{x^2}{\sqrt{x+3}}$; $x + 3 > 0$, so the domain is $(-3, \infty)$.

$) = \dfrac{\sqrt{x-2}}{\sqrt{x+4}} = \sqrt{\dfrac{x-2}{x+4}}$; $x - 2 \geq 0$ and $x + 4 > 0$, so $x \geq 2$ and $x > -4$; the domain is $[2, \infty$

$) = \dfrac{\sqrt{x+4}}{\sqrt{x-2}} = \sqrt{\dfrac{x+4}{x-2}}$; $x - 2 > 0$ and $x + 4 \geq 0$, so $x > 2$ and $x \geq -4$; the domain is $(2, \infty$

12. $f(x) = x^2 - 1;\ g(x) = 2x - 3$ 8; 3

13. $f(x) = x^2 + 4;\ g(x) = \sqrt{x + 1}$ 8; 3

In Exercises 15–22, find $f(g(x))$ and $g(f(x))$. State the domain of each.

15. $f(x) = 3x + 2;\ g(x) = x - 1$

15. $f(g(x)) = 3x - 1;\ (-\infty, \infty);\ g(f(x)) = 3x + 1;\ (-\infty, \infty)$

18. $f(x) = \dfrac{1}{x - 1};\ g(x) = \sqrt{x}$

18. $f(g(x)) = \dfrac{1}{\sqrt{x} - 1};\ [0, 1) \cup (1, \infty);\ g(f(x)) = \dfrac{1}{\sqrt{x - 1}};\ (1, \infty)$

19. $f(x) = x^2;\ g(x) = \sqrt{1 - x^2}$

19. $f(g(x)) = 1 - x^2;\ [-1, 1];\ g(f(x)) = \sqrt{1 - x^4};\ [-1, 1]$

24. $y = (x^3 + 1)^2$

26. $y = \dfrac{1}{x^3 - 5x + 3}$

24. One possibility: $f(x) = (x + 1)^2$ and $g(x) = x^3$

26. One possibility: $f(x) = 1/x$ and $g(x) = x^3 - 5x + 3$

32. A Snowball's Chance Jake stores a small cache of 4-inch diameter snowballs in the basement freezer, unaware that the freezer's self-defrosting feature will cause each snowball to lose about 1 cubic inch of volume every 40 days. He remembers them a year later (call it 360 days) and goes to retrieve them. What is their diameter then? 3.6 in.

The Inverse of a Function

The inverse of a function can be found by switching the domain (x) and the range (y).

Let's say set $A = \{(-1,0), (7,2), (4,-2)\}$

Domain: -1, 7, 4
Range: 0, 2, -2

Let's say set *A* = {(-1,0), (7,2), (4,-2)}

Domain: -1, 7, 4
Range: 0, 2, -2

If set *B* is the inverse of set *A*, then
set *B* = {(0,-1), (2,7), (-2,4)}

Domain: 0, 2, -2
Range: -1, 7, 4

If the original function is represented by *f(x)*, then we denote the inverse as

$$f^{-1}(x)$$

Determine the inverse of
$f(x) = 2x - 3$

STEP 1 SWITCH X's AND Y's

FIND $F^{-1}(X)$

$$x = 2y - 3$$

$$x + 3 = 2y$$

$$\frac{x+3}{2} = y$$

X	Y
2	1
3	3

Y	X
1	2
3	3

Determine the inverse of

$$f(x) = \frac{4x + 1}{2x - 5}$$

$$x = \frac{4y+1}{2y-5}$$

$$(2y-5)(x) = 4y+1$$

$$2xy - 5x = 4y + 1$$

$$2xy - 4y = 5x + 1$$

$$y(2x-4) = 5x+1$$

$$y = \frac{5x+1}{2x-4}$$

Determine the inverse of

$$f(x) = \frac{-2x - 7}{x + 3}$$

$$x = \frac{-2y-7}{y+3}$$

$$xy+3x = -2y-7$$

$$xy+2y = -3x-7$$

$$y(x+2 = \frac{-3x-7}{x+2}$$

Verify that the two functions $f(x) = 2x - 3$ and $f^{-1}(x) = \frac{x+3}{2}$ are inverses by showing both $f\left(f^{-1}(x)\right)$ and $f^{-1}\left(f(x)\right)$.

g(x)

$f(g(x))$

$f\left(\frac{x+3}{2}\right)$

$2\left(\frac{x+3}{2}\right) - 3$

$x + 3 - 3$

x

$g(f(x))$

$g(2x-3)$

$\frac{(2x-3)+3}{2}$

$\frac{2x}{2} = x$

Guided Practice:

1) Find the inverse of

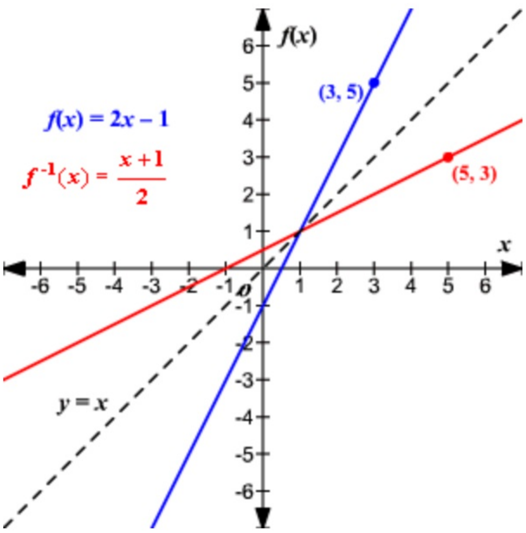
$$f(x)=\frac{3x-2}{5}$$

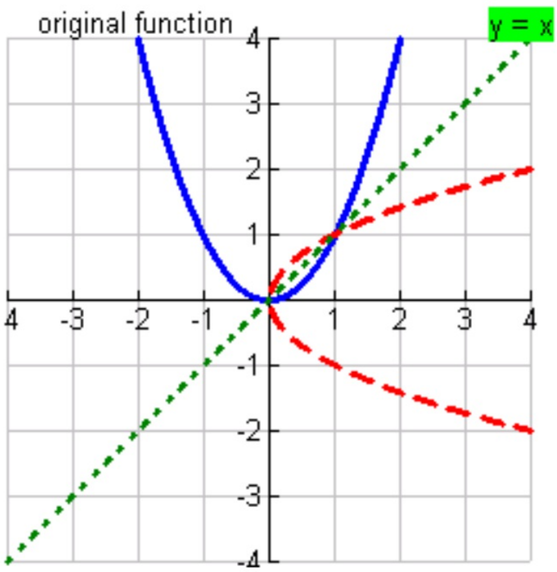
$$f^{-1}(x)=\frac{5x+2}{3}$$

2) Show that the functions are inverses of each other

$$f(x)=2x^3-1 \qquad g(x)=\sqrt[3]{\frac{x+1}{2}}$$

Graph of an Inverse Function





Reflection over the line *y* = *x*

$f(x) = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$f(x)$

x	y
-2	4
-1	1
0	0
1	1
2	4

$f^{-1}(x)$

x	y
4	-2
1	-1
0	0
1	1
4	2

f(x)

x	*y*
-2	4
-1	1
0	0
1	1
2	4

f $^{-1}$ *(x)*

x	*y*
4	-2
1	-1
0	0
1	1
4	2

Notice that while the original set is a function, the inverse is not a function (certain *x*-values produce two different *y*-values) *f(x)*

x	y
-2	4
-1	1
0	0
1	1
2	4

$f^{-1}(x)$

x	y
4	-2
1	-1
0	0
1	1
4	2

Not all functions have an inverse that is a function.

One-to-One

A function is one-to-one if *f(x)* has an inverse function $f^{-1}(x)$

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A function is one-to-one if *f(x)* has an inverse function $f^{-1}(x)$

Horizontal Line Test

The function has an inverse that is also a function if and only if any horizontal line can drawn such that it intersects the graph only once.

Vertical Line Test

Determines whether the equation itself is a function

Horizontal Line Test

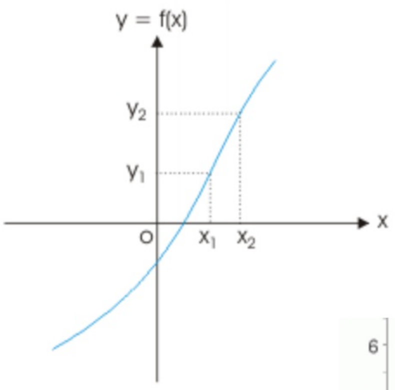
Determines whether the inverse will be a function

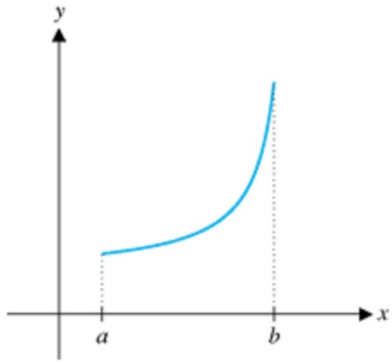
One-to-One

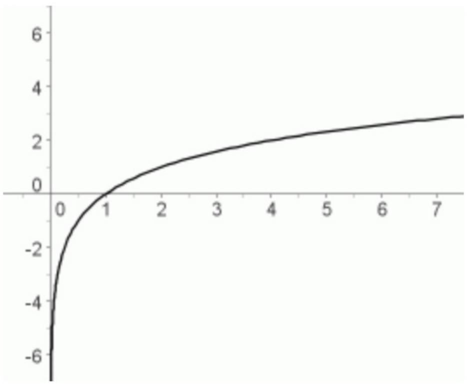
If a graph passes both the vertical and horizontal line test

Two cases that pass the horizontal line test:

1) If *f* is increasing on its entire domain

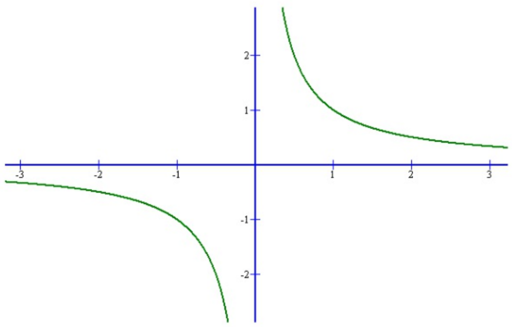
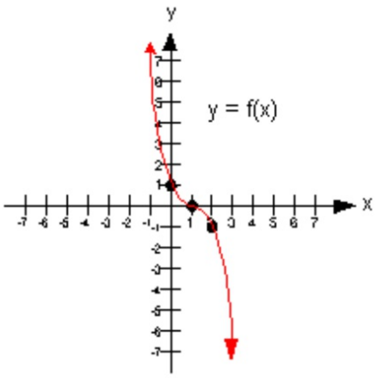


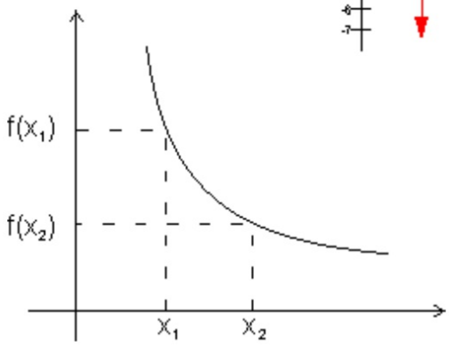




Two cases that pass the horizontal line test:

2) If *f* is decreasing on its entire domain



Exit Ticket (simplify all answers)

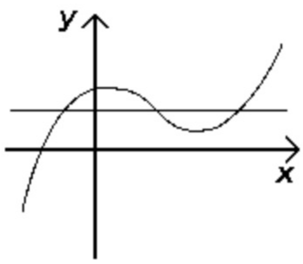
$f(x) = 4x^2$ & $g(x) = 3 - x$ & $h(x) = \sqrt{x} + 2$

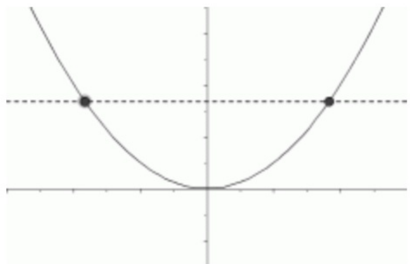
1) Find (h(f(x))

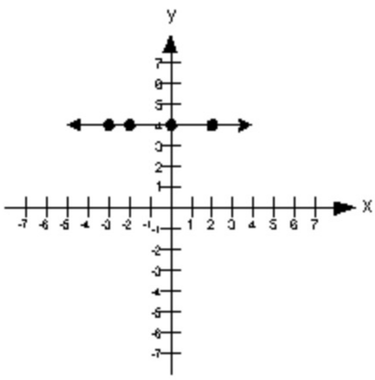
2) Find (g(h(1))

3) What is the inverse of f(x)?

4) What is $h^{-1}(x)$


y
x

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