Warm up

1. Graph the following function, including all asymptotes and holes

$$f(x) = \frac{x^2-4x-5}{x-5} \quad (x+1)(x-5)$$

2) Graph the Piecewise function

$$f(x) = \begin{cases} x^3, & x < 0 \\ 3x+2, & x \geq 0 \end{cases}$$

3) Solve for m:

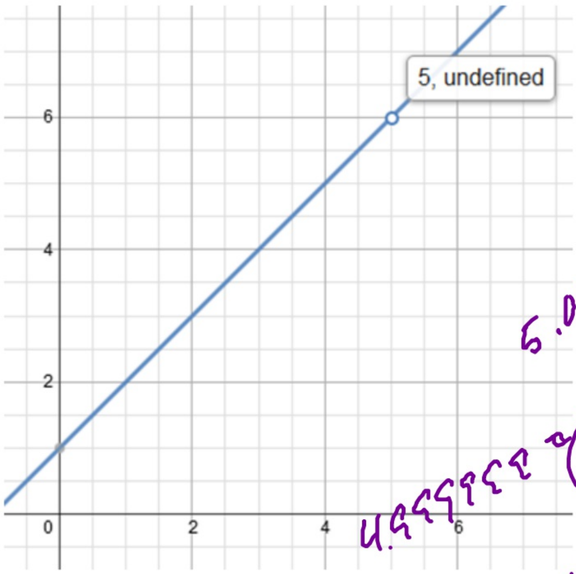
$$\frac{2m}{m-1} + \frac{m-5}{\cancel{m^2-1}} = 1 \quad (m+1)(m-1)$$

$$2m(m+1) + m - 5 = m^2 - 1$$

$$2m^2 + 2m + m - 5 - m^2 + 1 = 0$$

$$m^2 + 3m - 4 = 0$$

$$m = \cancel{1}, -4$$

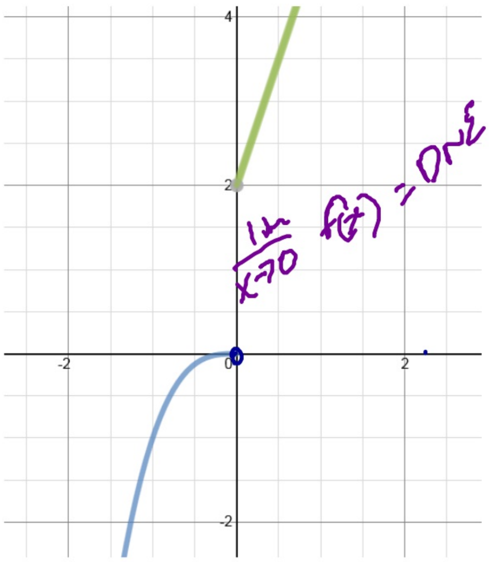


$$\lim_{x \to 5} f(x) = 6$$

5.0000001

4.999999

$$f(5) = \text{UND}$$



$$\lim_{x \to 0} f(x) = \text{DNE}$$

FINDING SUMS For part (a), find the sum of the first n terms of the arithmetic series. For part (b), find n for the given sum S_n.

45. $3 + 8 + 13 + 18 + 23 + \cdots$

~~**a.** $n = 20$~~ **b.** $S_n = 366$

46. $50 + 42 + 34 + 26 + 18 + \cdots$

a. $n = 40$ **b.** $S_n = 182$

47. $-10 + (-5) + 0 + 5 + 10 + \cdots$

~~**a.** $n = 19$~~ **b.** $S_n = 375$

48. $34 + 31 + 28 + 25 + 22 + \cdots$

a. $n = 32$ **b.** $S_n = -12$

49. $2 + 9 + 16 + 23 + 30 + \cdots$

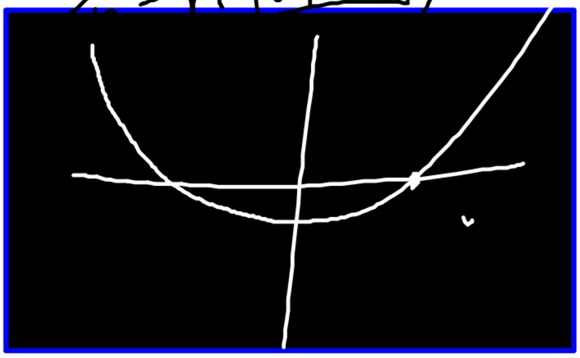
~~**a.** $n = 68$~~ **b.** $S_n = 1661$

45. a. 1010 **b.** 12 **47. a.** 665 **b.** 15
49. a. 16,082 **b.** 22 **51.** 1110

$a_n = 3 + (n-1)5$

$a_n = 5n - 2$

$S_n = n\,(a_1 + a_n)$



Objective: Define and find a simple limit

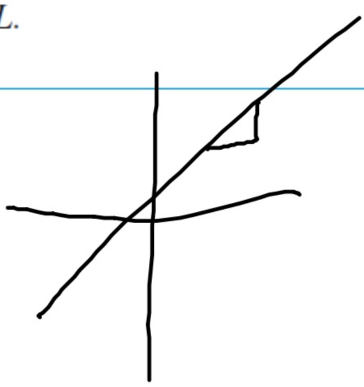
Definition of Limit

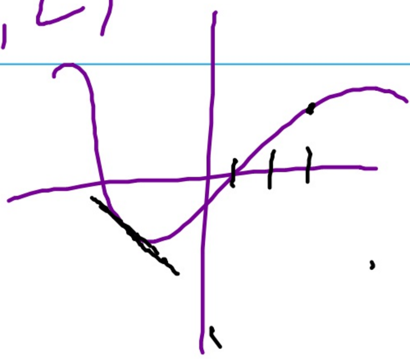
Definition of Limit

If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, the limit of $f(x)$ as x approaches c is L. This is written as

$$\lim_{x \to c} f(x) = L.$$

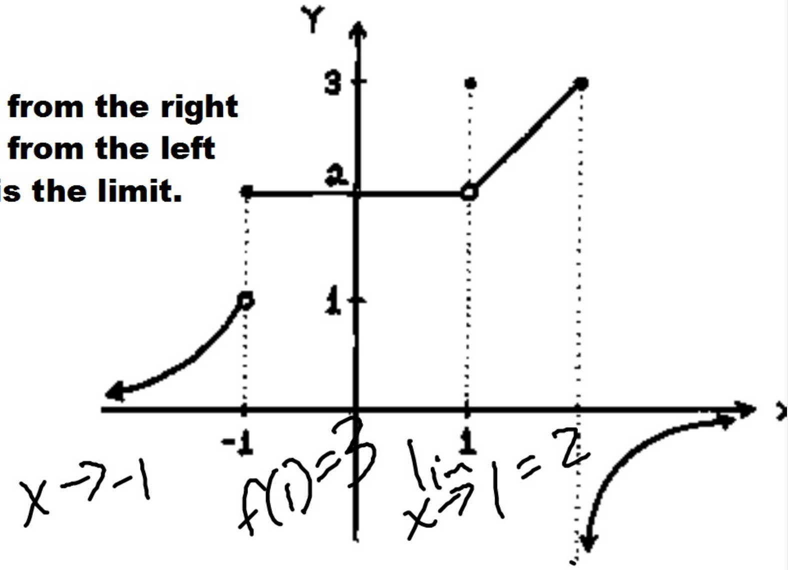
(c, L)





What is happening:

- Y value as x gets close to c from the right
- Y value as x gets close to c from the left
- If these values match, this is the limit.

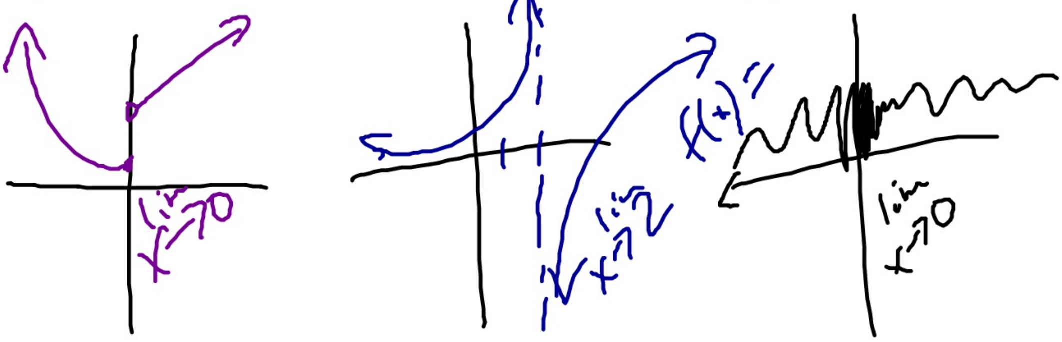


The Limit is always a number or it does not exist (DNE).

Conditions Under Which Limits Do Not Exist

The limit of $f(x)$ as $x \to c$ does not exist if any of the following conditions are true.

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side of c.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.



How to find a limit:

1. **Estimating by tables**

2. **Direct substitution**

3. **Dividing out/factoring**

4. **Rationalizing**

Direct Substitution

3) $\lim_{x \to 2} \left(\underset{2}{x^3} - \underset{2}{x^2} - 4\right)$

0

[redacted]

6) $\lim_{x \to \frac{3}{2}} -\sqrt{2x+4}$

$-\sqrt{7}$

[redacted]

$\lim_{x \to \pi} \sin(x)$

0

[redacted]

$$\lim_{x\to 1} - \frac{x-4}{x^2-6x+8}$$

[redacted]

But what about this one ?!?!

$$\lim_{x\to 2} - \frac{x^2-x-2}{x-2}$$

[redacted]

If you get 1/0 **, then you must try an algebraic method like:**

1. **Factoring**
2. **Rationalizing**
3. **Simplifying complex fractions**

Factoring

$$(\cancel{x+5})(x-2)$$

$$\lim_{x\to -5} \frac{x^2+3x-10}{\cancel{x+5}}$$

-7

[redacted]

$$(\cancel{x-2})(x+1)$$

$$\lim_{x\to 2} - \frac{x^2-x-2}{\cancel{x-2}}$$

[redacted]

-3

$$\lim_{x\to 1} - \frac{x^2-1}{x-1}$$

[redacted]

-2

Rationalizing (I think I hear screaming)

$$\lim_{x\to 5}\frac{x-5}{\sqrt{x+4}-3}\cdot\frac{\sqrt{x+4}+3}{(\sqrt{x+4}+3)}$$

6

$$\frac{\cancel{x-5}(\sqrt{x+4}+3)}{\cancel{x+4-9}}$$

$$\cancel{(x-5)}$$

[redacted]

$$\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x-3}\cdot\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}$$

$\frac{1}{6}$

$$\frac{\cancel{(x-3)}}{\cancel{x+6-9}}\quad 1$$

$$\cancel{x-3}(\sqrt{x+6}+3)$$

[redacted]

Simplifying complex fractions

[redacted]

$$\lim_{x\to -3}\frac{x}{\frac{1}{3+x}-\frac{1}{3}}\cdot\frac{3(3+x)}{3(3+x)}$$

0

$$\frac{3x(3+x)}{3-(3+x)}\quad -3$$

$-3x$

[redacted]

$$\lim_{x\to 0}\frac{\frac{1}{-4+x}+\frac{1}{4}}{x}\cdot\frac{4(x-4)}{4(x-4)}$$

$-\frac{1}{16}$

$$\frac{1}{4(x-4)}$$

$$\frac{4+\cancel{x}-4}{4x(x-4)}\quad 1$$

Estimating by tables (if nothing else works)

$$f(x) = \frac{x^3 - 1}{x - 1} \qquad \text{Find } \lim_{x \to 1} f(x) = 3$$

X	.9	.99	.99999	1	1.00001	1.01	1.1
Y	2.71	2.97	2.9997	UND.	3.00003	3.03	3.3

Estimating by tables

$$f(x) = \frac{x}{x - 3} \qquad \text{Find } \lim_{x \to 3} f(x)$$

	3	
-299	UND	3001

PROBLEMS

1. Use the graph in Fig. 10 to determine the following limits.

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 2} f(x)$

(c) $\lim_{x\to 3} f(x)$ (d) $\lim_{x\to 4} f(x)$

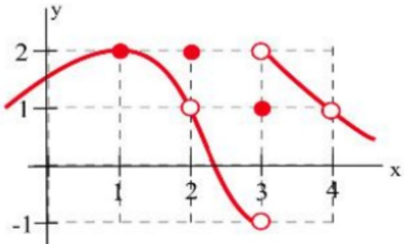


Fig. 10

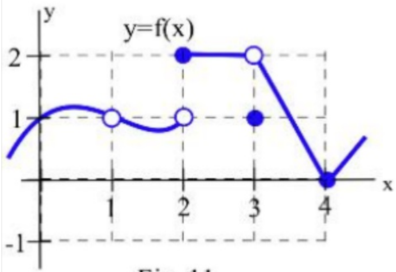


Fig. 11

2. Use the graph in Fig. 11 to determine the following limits.

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 2} f(x)$

(c) $\lim_{x\to 3} f(x)$ (d) $\lim_{x\to 4} f(x)$

WARM UP

Properties of limits

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = K$$

1. Scalar multiple: $\lim_{x\to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x\to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x\to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x\to c} [f(x)]^n = L^n$

Use properties of limits to evaluate

1. $\lim_{x\to c} f(x) = 3, \quad \lim_{x\to c} g(x) = 6$

(a) $\lim_{x\to c} [-2g(x)]$ (b) $\lim_{x\to c} [f(x) + g(x)]$

(c) $\lim_{x\to c} \frac{f(x)}{g(x)}$ (d) $\lim_{x\to c} \sqrt{f(x)}$

---

2. $\lim_{x\to c} f(x) = 5, \quad \lim_{x\to c} g(x) = -2$

(a) $\lim_{x\to c} [f(x) + g(x)]^2$ (b) $\lim_{x\to c} [6f(x)g(x)]$

(c) $\lim_{x\to c} \frac{5g(x)}{4f(x)}$ (d) $\lim_{x\to c} \frac{1}{\sqrt{f(x)}}$

One sided Limits

$$\lim_{x\to -3^-} \frac{2x}{x+3}$$

[redacted]

$$\lim_{x\to 1} -\frac{3}{x-1}$$

[redacted]

$$\lim_{x\to -2^+} \frac{1}{x^2-4}$$

[redacted]

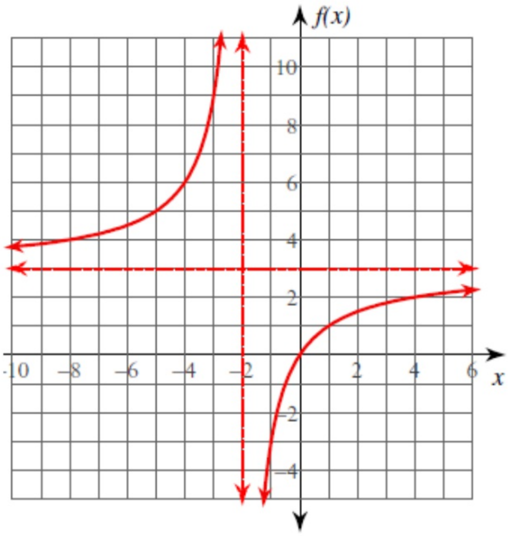
$$\lim_{x\to 3^-} -\frac{4x}{x-3}$$

[redacted]

$$\lim_{x\to -3^-} -\frac{2}{x+3}$$

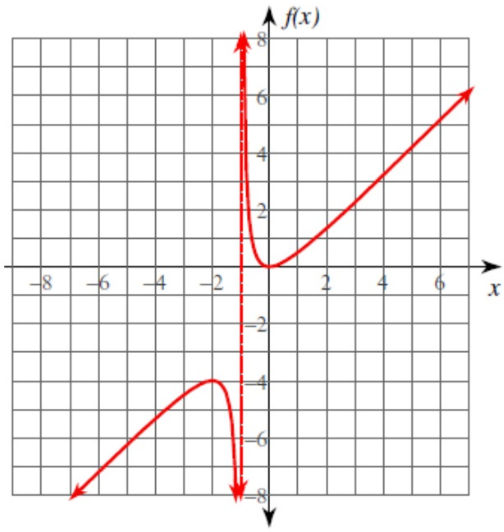
[redacted]

$$\lim_{x\to -2^+} \frac{3x}{x+2}$$



[redacted]

$$\lim_{x\to -1^+} \frac{x^2}{x+1}$$



[redacted]

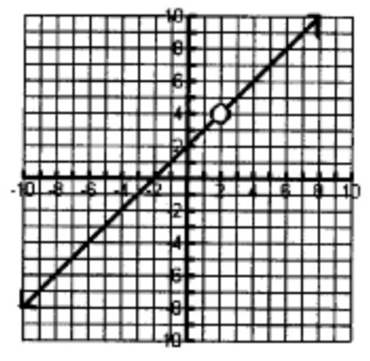
One Sided Limits

1 a) Find the $\lim_{x\to 1} f(x)$

b) Find the $\lim_{x\to 2^+} f(x)$

c) Find the $\lim_{x\to 2^-} f(x)$

d) Find the $\lim_{x\to 2} f(x)$



2. $$f(x) = \begin{cases} x^2+2 & x>0 \\ 2-2x & x<0 \\ 2 & x=0 \end{cases}$$

a) Find the $\lim_{x\to 0^+} f(x)$

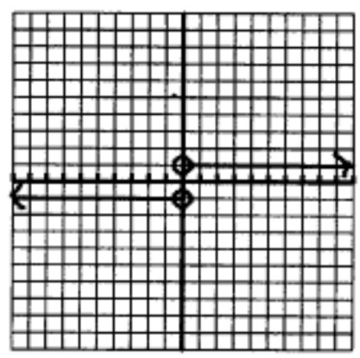
b) Find the $\lim_{x\to 0^-} f(x)$

c) Find the $\lim_{x\to 0} f(x)$

3) a) Find the $\lim_{x\to 0^+} f(x)$

b) Find the $\lim_{x\to 0^-} f(x)$

c) Find the $\lim_{x\to 0} f(x)$



4) a) Find the $\lim_{x\to 3^+} f(x)$

b) Find the $\lim_{x\to 3^-} f(x)$

c) Find the $\lim_{x\to 3} f(x)$