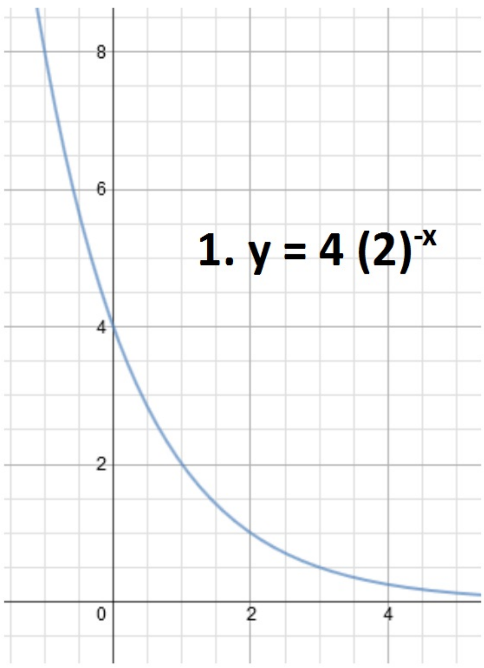
Warm up

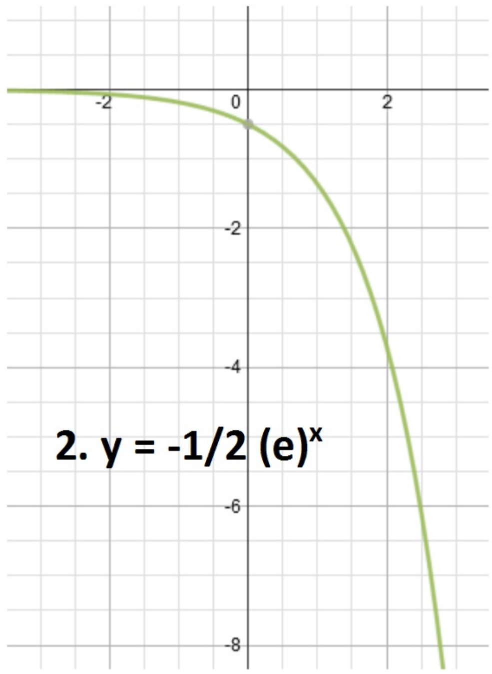
Graph the following and list the key point and range

1. $y = 4 (2)^{-x}$

2. $y = -1/2 (e)^{x}$

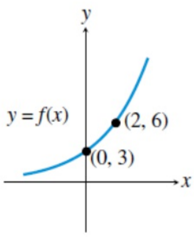
3. Find the exponential function given the points (0, 3) and (2, 0.12)


1. $y = 4\ (2)^{-x}$
8
6
4
2
0
2
4

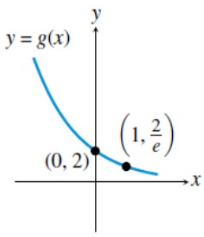

2. $y = -1/2\ (e)^{x}$
-2
0
2
-2
-4
-6
-8

In Exercises 13 and 14, determine a formula for the exponential function whose graph is shown in the figure.

13. $f(x)$ $3 \cdot 2^{x/2}$



14. $g(x)$ $2e^{-x}$



In Exercises 15–24, describe how to transform the graph of f into the graph of g. Sketch the graphs by hand and support your answer with a grapher.

15. $f(x) = 2^x$, $g(x) = 2^{x-3}$ Translate $f(x) = 2^x$ by 3 units to the right.

16. $f(x) = 3^x$, $g(x) = 3^{x+4}$ Translate $f(x) = 3^x$ by 4 units to the left.

24. $f(x) = e^x$, $g(x) = 3e^{2x} - 1$

24. Horizontally shrink $f(x) = e^x$ by a factor of 2, vertically stretch by a factor of 3, and shift down one unit.

In Exercises 31–34, state whether the function is an exponential growth function or exponential decay function, and describe its end behavior using limits.

31. $f(x) = 3^{-2x}$

32. $f(x) = \left(\frac{1}{e}\right)^x$ Exponential decay; $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to -\infty} f(x) = \infty$

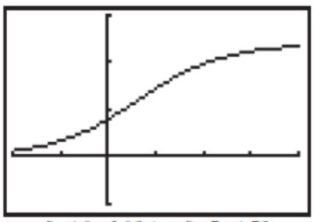
33. $f(x) = 0.5^x$ Exponential decay; $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to -\infty} f(x) = \infty$

In Exercises 41–44, use a grapher to graph the function. Find the y-intercept and the horizontal asymptotes.

41. $f(x) = \dfrac{12}{1 + 2 \cdot 0.8^x}$

42. $f(x) = \dfrac{18}{1 + 5 \cdot 0.2^x}$

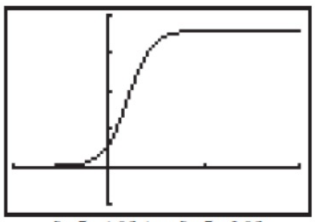
41.



[–10, 20] by [–5, 15]

y-intercept: (0, 4)
Horizontal asymptotes:
$y = 0$, $y = 12$

42.



[–5, 10] by [–5, 20]

y-intercept: (0, 3)
Horizontal asymptotes:
$y = 0$, $y = 18$

Objective:

- Graph log functions with transformations
- Use properties of log functions

http://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/

How would you graph $y=\log_2 x$?

☆Just switch x and y in the above table since $y=2^x$ and $y = \log_2 x$ are ___inverses___.

(0,1)

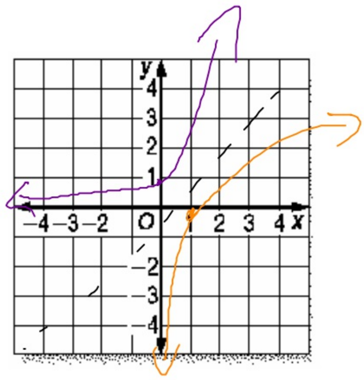
$y = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

(1,0)

☆ $y = \log_2 x$

x	y
1/8	-3
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

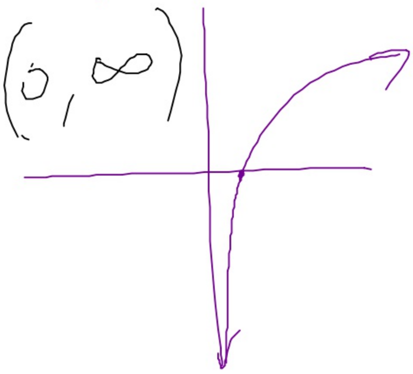


Graph Activity

Graph y=ln (x)

$\log_6 x$ $\log x$

$\log_5 x$ $\log_2 x$



ln (x) + 2 SHIFT ↑2

ln (x) + 2

ln (x + 2) SHIFT ←

ln (x - 2) →

2 ln (x) VERT STRETCH

- ln (x) FLIP OVER X

ln (-x) FLIP Y

ln (2x) HORIZ. STRETCH

$\log_{10} X$

Common log

$\ln x = \log_e X$

What is a logarithmic function?

bae
base
answer
exponent

☆ **The ________________ of an exponential function.**

Suppose $b > 0$ and $b \neq 1$. For $a > 0$, there is a number p such that if and only if $b^p = a$, then $\log_b a = p$.

$\log_b a = p$

b a e

$b^p = a$

$b^p = a$

What is logarithmic form?

What is exponential form?

GUIDED PRACTICE:

Write each equation in exponential form.

1. $\log_3 9 = 2$ $\quad 3^2 = 9$

   b a e

2. $\log_8 1 = 0$ $\quad 8^0 = 1$

3. $\log_{10} \frac{1}{100} = -2$ $\quad 10^{-2} = \frac{1}{100}$

4. $\log_{216} 6 = \frac{1}{3}$ $\quad 216^{\frac{1}{3}} = 6$

INDEPENDENT PRACTICE:

GUIDED PRACTICE:
Write each equation in logarithmic form.

5. $5^3 = 125$

6. $27^{\frac{1}{3}} = 3$

$\log_5 125 = 3$

$\log_{27} 3 = 1/3$

Basic properties of logs

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

$b^{\log_b x}$

Examples

$\log_9 9^2 = 2$

$7^{\log_7 (x^2 - 1)} = x^2 - 1$

What are the
Properties of Logarithms?

- **Product Property** ☆ $\log_b m + \log_b n = \log_b mn$
- **Quotient Property** ☆ $\log_b m - \log_b n = \log_b \frac{m}{n}$
- **Power Property** ☆ $n\log_b m = \log_b m^n$ (power up!)

☆ **This is called __condensing__ the logarithms:**

EX. $\log_2 3 + \log_2 5 = \log_2 15$

☆ **In reverse, this is called __expanding__ the logarithms:**

EX. $\log_2 21 = \log_2 (3)(7) = \log_2 3 + \log_2 7$

Expand the following logs

$\log_6 \frac{x-1}{y^2}$

$\log_6 x-1 - \log_6 y^2$

$\log_6 x-1 - 2\log_6 y$

$\ln\sqrt{x}\left(y^3\right)$

$\ln x^{1/2} + \ln y^3$

$\frac{1}{2}\ln x + 3\ln y$

Condense the following logs

$\log_2 3 + \log_2 x^4$

$\log_2 3x^4$

$3\log_5(x-1) - 2\log_5(3)$

$\log_5 \frac{(x-1)^3}{3^2 \text{ or } 9}$

- **Change of Base Formula:** For all positive numbers a and b, where $b \neq 1$:

$$\log_b a = \frac{\log a}{\log b}$$

GUIDED PRACTICE

Express each logarithm in terms of common logarithm. Then approximate its value to four decimal places.

9. $\log_{25} 4$ $\frac{\log 4}{\log 25}$ 0.4307 $\frac{\ln 4}{\ln 25}$ MATH → ALPHA → MATH

10. $\log_6 10$

11. $\log_7 3$

12. $\log_2 35$

Wrap up

1. Expand $\log_4 \sqrt[6]{\dfrac{x^7 y^3}{z^8}}$

2. Graph $\log_2(x+7)$

3. Tell if growth or decay and give the rate

$a.\ f(x) = 100(.92)^x \qquad b.\ f(x) = 500(1.095)^x$