Warm-up
Prove the following Trig Identities

1. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$= 1$

2. $\dfrac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

3. $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

Warm-up

$$= \tan x \sin x + \cos x = \frac{\sin x}{\cos x} \cdot \sin x + \cos x = \frac{\sin^2 x}{\cos x} + \cos x = \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \text{RHS}$$

2.

$$\frac{\tan^2 x}{\tan^2 x + 1} = \frac{\left(\frac{\sin x}{\cos x}\right)^2}{\left(\frac{\sin x}{\cos x}\right)^2 + 1} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + 1} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}}$$

$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x = \text{RHS}$$

3.

$$= \frac{\cos x}{1 - \sin x} = \frac{\cos x}{1 - \sin x} \cdot 1 = \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x\,(1 + \sin x)}{(1 - \sin x)\,(1 + \sin x)}$$

$$= \frac{\cos x\,(1 + \sin x)}{1 - \sin^2 x} = \frac{\cos x\,(1 + \sin x)}{\cos^2 x} = \frac{1 + \sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \text{LHS}$$

More practice...

Work on Worksheet #2

Numbers 9, 11, 13, 15, 19, 23

Which expression is equivalent to $(\sec\theta)\left(\frac{\sin\theta}{\tan\theta}\right)$?

A $\cos^2\theta - \sin^2\theta$

B $\sin^2\theta - \cos^2\theta$

C $\cot^2\theta - \csc^2\theta$

D $\csc^2\theta - \cot^2\theta$

$= 1$

$\cot^2 x + 1 = \csc^2 x$

Objective: Solve trig equations

Simple equations

1. **Isolate the trig functions**
2. **Find all solutions in the given interval** $[0, 2\pi]$
3. **If not given an interval the answer should be in the form of a formula**

$x = \frac{\pi}{a} + \pi n$

Examples:

1. 2 sin x = 1 When $0 < x < \pi$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

2. Sin x + Sin x $= \sqrt{3}$ When $0 < x < 2\pi$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}$

Examples:

3. tan x + 4 = 5

$\tan x = 1$

$x = \frac{\pi}{4} + 2\pi n \quad \frac{9\pi}{4}$

$\frac{5\pi}{4} + 2\pi n \quad \frac{13\pi}{4}$

$x = \frac{\pi}{4} + \pi n$

$\sqrt{x^2} = \sqrt{4}$

± 2

4. $3\sec^2 x - 4 = 0$

$\sqrt{\sec^2 x} = \sqrt{\frac{4}{3}}$

$\sec x = \frac{\pm 2}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$

$x = \frac{5\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$

$\frac{\pi}{6} + \pi n$

$\frac{5\pi}{6} + \pi n$

Worksheet
Supplement to Chapter 5

Numbers 1, 8, 10, 21, 22

$\cos^2 x - 1 \neq \sin^2 x$

Solve trig equations with factoring

1. **Set = 0**
2. **First take out GCF**
3. **Can (you) factor what is left?**

Find the values of x in $[0, 2\pi)$

1. $2\sin^2 x + \sin x = 1$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = -1$

$x = \frac{3\pi}{2}$

2. $\sqrt{2}\tan x \cos x - \tan x = 0$

$\tan x(\sqrt{2}\cos x - 1) = 0$

$\tan x = 0$

$x = 0, \pi$

$\sqrt{2}\cos x - 1 = 0$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

Practice worksheet 5, 11, 12, 15, 16, 21, 22, 34, 35