# Warm up

Find the inverse of the following functions. Give the domain

1. $f(x) = \sqrt{x-4}$

2. $f(x) = \frac{2x+3}{x-5}$

Verify if the following functions are inverses by composites

3. $f(x) = x^2 - 4$ when $x \geq 0$ $\quad g(x) = \sqrt{x+4}$

4. If $f(x) = \begin{cases} ax + 6 & x > 2 \\ ax^2 + 2 & x \leq 2 \end{cases}$ What value of ***a*** will make this functions continious?

**14.** $y = 2x + 5 \Rightarrow$

$$x = 2y + 5$$
$$2y = x - 5$$
$$f^{-1}(x) = y = \frac{x-5}{2} = \frac{1}{2}x - \frac{5}{2};$$
$$(-\infty, \infty)$$

**16.** $y = \dfrac{x+3}{x-2} \Rightarrow$

$$x = \frac{y+3}{y-2}$$
$$x(y-2) = y + 3$$
$$xy - 2x = y + 3$$
$$xy - y = 2x + 3$$
$$y(x-1) = 2x + 3$$
$$f^{-1}(x) = y = \frac{2x+3}{x-1};$$
$$x \neq 1 \text{ or } (-\infty, 1) \cup (1, \infty)$$

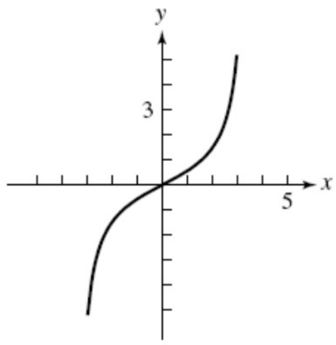
**19.** $y = x^3 \Rightarrow$

$$x = y^3$$
$$f^{-1}(x) = y = \sqrt[3]{x}\ ; (-\infty, \infty)$$

**22.** $y = \sqrt[3]{x-2} \Rightarrow$

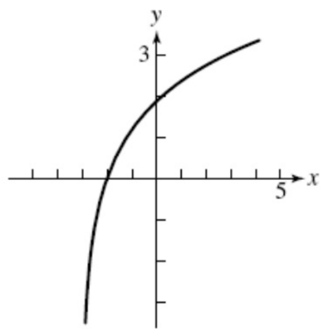
$$x = \sqrt[3]{y-2}$$
$$x^3 = y - 2$$
$$f^{-1}(x) = y = x^3 + 2; (-\infty, \infty)$$

**23.** One-to-one




**24.** Not one-to-one

**25.** One-to-one




**26.** Not one-to-one

**27.** $f(g(x)) = 3\left[\frac{1}{3}(x + 2)\right] - 2 = x + 2 - 2 = x;$

$g(f(x)) = \frac{1}{3}[(3x - 2) + 2] = \frac{1}{3}(3x) = x$

**28.** $f(g(x)) = \frac{1}{4}[(4x - 3) + 3] = \frac{1}{4}(4x) = x;$

$g(f(x)) = 4\left[\frac{1}{4}(x + 3)\right] - 3 = x + 3 - 3 = x$

**34. (a)** $9c(x) = 5(x - 32)$

$\frac{9}{5}c(x) = x - 32$

$\frac{9}{5}c(x) + 32 = x$

In this case, $c(x)$ becomes $x$, and $x$ becomes $c^{-1}(x)$ for the inverse. So, $c^{-1}(x) = \frac{9}{5}x + 32$. This converts Celsius temperature to Fahrenheit temperature.

**(b)** $(k \circ c)(x) = k(c(x)) = k\left(\frac{5}{9}(x - 32)\right)$

$\frac{5}{9}(x - 32) + 273.16 = \frac{5}{9}x + 255.38$. This is used to convert Fahrenheit temperature to Kelvin temperature.

# 2.2 Power Functions with Modeling

**Objective: Describe and create power functions**

Power function:

$y = k x^a$

*power*

FORM: $f(x)=k\cdot x^a$

*constant of variation*
*(constant of proportion)*

# Common Power Functions:

Circumference

$$C = 2\pi r$$

Area of a circle

$$A = \pi r^2$$

Force of gravity

$$F = M \times G$$

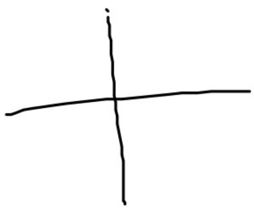
Boyle's Law

$$P = \frac{k}{V}$$

$$P = kV^{-1}$$

| | Power? | Constant of variation |
|---|---|---|
| Circumference | 1 | $2\pi$ |
| Area of a circle | 2 | $\pi$ |
| Force of gravity | 1 | $G$ |
| Boyle's Law | -1 | $k$ |

Analyzing Power Functions:



$f(x)= x^{-3}$ and $f(x)= x^{-5}$

$f(x)= x^{1/3}$ and $f(x)= x^{1/5}$

$f(x)= x^{-4}$ and $f(x)= x^{-6}$

$f(x)= x^{-1/4}$ and $f(x)= x^{-1/6}$

*Domain:*

*Range:*

*Continuity:*

*Even, odd, neither:*

Positive powers are statements of ______________________.

Negative powers are statements of ____________________.

**Four Steps to Solve a Variation Problem**

1. Write the general variation formula for the problem.
2. Use the formula to find the constant of variation, ***k***.
3. Rewrite the formula, including the value of ***k***.
4. Answer the question.

**Direct Variation:** $y = kx$

"$y$ varies directly as $x$"

"$y$ is directly proportional to $x$"

$k$ is the constant of variation

1. The number of centimeters $y$ in a linear measurement varies directly with the number of inches $x$ in the measurement. Pablo's height is 152.4 centimeters or 60 inches. What is Maria's height in centimeters if she is 64 inches tall?

m = kh
28.50 = k6
4.75 = k
→ m = 4.75h
m = 4.75(10)
$47.50

**Joint Variation:** $y = kxz$

"$y$ varies jointly as $x$ and $z$"

$k$ is the constant of variation

6. The cost $c$ of materials for a deck varies jointly with the width $w$ and the length $l$. If $c = \$470.40$ w hen $w = 12$ and $l = 16$, find the cost when $w = 10$ and $l = 25$.

$C = kwL$

$470.40 = k(12)(16)$

$2.45 = K$

$C = 2.45wL$

$C = 2.45(10)(25)$

\$612.50

**Inverse Variation:** $y = k / x$ indirectly varies

10) $t = \frac{21}{e}$

4.2 days

"$y$ varies inversely as $x$"

"$y$ is inversely proportional to $x$"

$k$ is the constant of variation

9. The volume $V$ of a gas kept at a constant temperature varies inversely as the pressure $p$. If the pressure is 24 pounds per square inch, the volume is 15 cubic feet. What will be the volume when the pressure is 30 pounds per square inch?

$V = \frac{k}{p}$

$15 = \frac{k}{24}$ $k = 360$

$V = \frac{360}{p}$

$V = \frac{360}{30}$

$V = 12 \text{ ft}^3$

14. To build a sound wall along the highway, the amount of time $t$ needed varies **directly** with the number of cement blocks $c$ needed and **inversely** with the number of workers $w$. A sound wall made of 2400 blocks, using six workers takes 18 hours to complete. How long would it take to build a wall of 4500 blocks with 10 workers? HINT: $t = \frac{kc}{w}$

$$t = \frac{kC}{W} \quad t = \frac{.045\ C}{W}$$

20.25 hrs.

**57. Light Intensity** Velma and Reggie gathered the data in Table 2.13 using a 100-watt light bulb and a Calculator-Based Laboratory$^{TM}$ (CBL$^{TM}$) with a light-intensity probe.

**(a)** Draw a scatter plot of the data in Table 2.13

**(b)** Find the power regression model. Is the power close to the theoretical value of $a = -2$? [redacted]

**(c)** Superimpose the regression curve on the scatter plot.

**(d)** Use the regression model to predict the light intensity at distances of 1.7m and 3.4 m.

**Table 2.13 Light Intensity Data for a 100-W Light Bulb**

| Distance (m) | Intensity ($W/m^2$) |
|---|---|
| 1.0 | 7.95 |
| 1.5 | 3.53 |
| 2.0 | 2.01 |
| 2.5 | 1.27 |
| 3.0 | 0.90 |

**34.** First divide $f(x)$ by $x - 4i$. Then divide the result, $x^3 + 4ix^2 - 3x - 12i$, by $x + 4i$. This leaves the polynomial $x^2 - 3$. Zeros: $x = \pm\sqrt{3}, x = \pm 4i$

| | | | | | |
|---|---|---|---|---|---|
| $4i$ | 1 | 0 | 13 | 0 | $-48$ |
| | | $4i$ | $-16$ | $-12i$ | 48 |
| | 1 | $4i$ | $-3$ | $-12i$ | 0 |

| | | | | |
|---|---|---|---|---|
| $-4i$ | 1 | $4i$ | $-3$ | $-12i$ |
| | | $-4i$ | 0 | $12i$ |
| | 1 | 0 | $-3$ | 0 |

$f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 4i)(x + 4i)$

**39.** $f(x) = (x - 1)(2x^2 + x + 3)$

| | | | | |
|---|---|---|---|---|
| 1 | 2 | $-1$ | 2 | $-3$ |
| | | 2 | 1 | 3 |
| | 2 | 1 | 3 | 0 |

**6.** $(x - 1 + 2i)(x - 1 - 2i) = x^2 - 2x + 5$

**10.** $(x + 1)(x - 2)(x - 1 + i)(x - 1 - i)$
$= (x + 1)(x - 2)(x^2 - 2x + 2)$
$= x^4 - 3x^3 + 2x^2 + 2x - 4$

**14.** $(x + 1)^3(x - 3) = x^4 - 6x^2$

**22.** 3 complex zeros; all 3 real.

**23.** 3 complex zeros; 1 real.

**28.** Zeros: $x = 3$ (graphically) and $x = \dfrac{7}{2} \pm \dfrac{\sqrt{43}}{2}i$ (applying the quadratic formula to $x^2 - 7x + 23$).

| | | | | |
|---|---|---|---|---|
| 3 | 1 | −10 | 4 | −69 |
| | | 3 | −21 | 69 |
| | 1 | −7 | 23 | 0 |

$f(x)$
$= (x - 3)\left[x - \left(\dfrac{7}{2} - \dfrac{\sqrt{43}}{2}i\right)\right]\left[x - \left(\dfrac{7}{2} + \dfrac{\sqrt{43}}{2}i\right)\right]$
$= \dfrac{1}{4}(x - 3)(2x - 7 + \sqrt{43}\,i)(2x - 7 - \sqrt{43}\,i)$

[redacted]

**33.** First divide $f(x)$ by $x - (1 + i)$ (synthetically). Then divide the result, $x^3 + (-1 + i)x^2 - 3x + (3 - 3i)$, by $x - (1 - i)$. This leaves the polynominal $x^2 - 3$.
Zeros: $x = \pm\sqrt{3}, x = 1 \pm i$

| | | | | | |
|---|---|---|---|---|---|
| $1 + i$ | $1$ | $-2$ | $-1$ | $6$ | $-6$ |
| | | $1 + i$ | $-2$ | $-3 - 3i$ | $6$ |
| | $1$ | $-1 + i$ | $-3$ | $3 - 3i$ | $0$ |

| | | | | |
|---|---|---|---|---|
| $1 - i$ | $1$ | $-1 + i$ | $-3$ | $3 - 3i$ |
| | | $1 - i$ | $0$ | $-3 + 3i$ |
| | $1$ | $0$ | $-3$ | $0$ |

$$f(x) = (x - \sqrt{3})(x + \sqrt{3})[x - (1 - i)][x - (1 + i)]$$
$$= (x - \sqrt{3})(x + \sqrt{3})(x - 1 + i)(x - 1 - i)$$

**40.** $f(x) = (x - 1)(3x^2 + x + 2)$

| | | | | |
|---|---|---|---|---|
| 1 | 3 | −2 | 1 | −2 |
| | | 3 | 1 | 2 |
| | 3 | 1 | 2 | 0 |

**55.** Both the sum and the product of two complex conjugates are real numbers, and the absolute value of a complex number is always real. The square of a complex number, on the other hand, need not be real. The answer is E.

**56.** Allowing for multiplicities other than 1, then, the polynomial can have anywhere from 1 to 5 distinct real zeros. But it cannot have no real zeros at all. The answer is A.

**57.** Because the complex, non-real zeros of a real-coefficient polynomial always come in conjugate pairs, a polynomial of degree 5 can have either 0, 2, or 4 non-real zeros. The answer is C.

**58.** A polynomial with real coefficients can never have an odd number of non-real complex zeros. The answer is E.