Warm up

Work on 31-39 odds on HW worksheet

4-28 by 4's
32-40 evens

Homework and Warm up

Answers to Limits Practice

1) -1
2) 2
3) 2
4) 2
5) 2
6) 2
7) -5
8) -4
9) 0
10) 5
11) -7
12) 0
13) 2
14) 0
15) $\frac{11}{2}$
16) -2
17) -3
18) $-\frac{1}{2}$
19) 3
20) 0
21) 0
22) $\frac{5}{2}$
23) 1
24) 5
25) $\frac{1}{12}$
26) 3
27) $\frac{6}{25}$
28) $\frac{50}{29}$
29) 1
30) $\frac{1}{5}$
31) -3
32) $-\frac{1}{4}$
33) -2
34) $\frac{1}{2}$
35) -1
36) -1
37) 4
38) $\frac{1}{2}$
39) Does not exist.
40) Does not exist.
41) 5
42) -2

ROBLEMS

Use the graph in Fig. 10 to determine the following limits.

(a) $\lim_{x\to 1} f(x)$ 2 (b) $\lim_{x\to 2} f(x)$ 1

(c) $\lim_{x\to 3} f(x)$ DNE (d) $\lim_{x\to 4} f(x)$ 1

f(4) = und. f(2) = 2

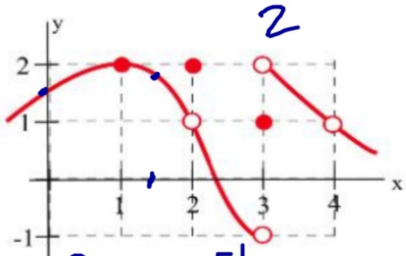


Fig. 10

1.99999
2.00001

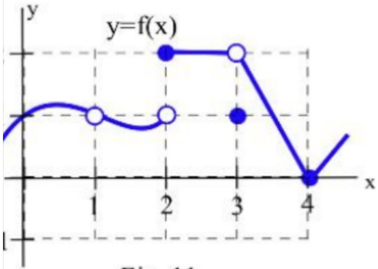


Fig. 11

2. Use the graph in Fig. 11 to determine the following limits.

(a) $\lim_{x\to 1} f(x)$ 1 (b) $\lim_{x\to 2} f(x)$ DOES NOT EXIST

(c) $\lim_{x\to 3} f(x)$ 2 (d) $\lim_{x\to 4} f(x)$ 0

Properties of limits

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = K$$

1. Scalar multiple: $\lim_{x\to c} [b\,f(x)] = bL$
2. Sum or difference: $\lim_{x\to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x\to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x\to c} [f(x)]^n = L^n$

Use properties of limits to evaluate

1. $\lim_{x\to c} f(x) = 3, \quad \lim_{x\to c} g(x) = 6$

(a) $\lim_{x\to c} [-2g(x)]$ (b) $\lim_{x\to c} [f(x) + g(x)]$

(c) $\lim_{x\to c} \frac{f(x)}{g(x)}$ (d) $\lim_{x\to c} \sqrt{f(x)}$

2. $\lim_{x\to c} f(x) = 5, \quad \lim_{x\to c} g(x) = -2$

(a) $\lim_{x\to c} [f(x) + g(x)]^2$ (b) $\lim_{x\to c} [6f(x)g(x)]$

(c) $\lim_{x\to c} \frac{5g(x)}{4f(x)}$ (d) $\lim_{x\to c} \frac{1}{\sqrt{f(x)}}$

One-Sided Limits

Let $f(x) = \frac{x^3 - x}{x - 1}$.

What happens to the values of $f(x)$ as x approaches 1 **from the left** (i.e., x increases to 1)? $\lim_{x\to 1^-}$ 2

x	0	0.5	0.75	0.9	0.99	0.999
$f(x)$	0	0.75	1.3125	1.71	1.9701	1.997001

It looks like the values of $f(x)$ are approaching 2.

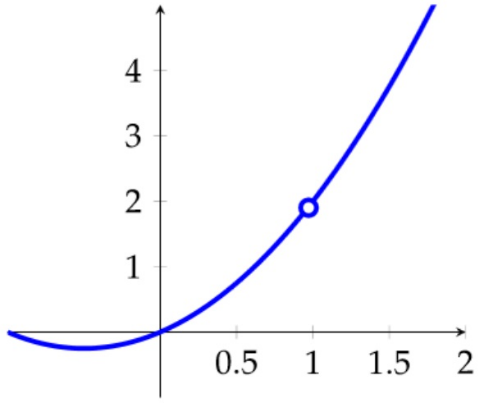
What happens to the values of $f(x)$ as x approaches 1 **from the right** (i.e., x decreases to 1)? $\lim_{x\to 1^+}$ 2

x	2	1.5	1.25	1.1	1.01	1.001
$f(x)$	6	3.75	2.8125	2.31	2.0301	2.003001

The values of $f(x)$ are approaching 2. $\lim_{x\to 1} f(x) = 2$

Let $f(x) = \dfrac{x^3 - x}{x - 1}$.

This is a bit easier to see graphically:



So, the value of $f(x)$ approaches 2 (*although it will never actually be equal to 2*) as x approaches 1 from either the right or the left.

Definition (intuitive)

Let f be a function and c a real number.

We say that a *finite* real number L is the **limit of f as x approaches c from the left** if the value of $f(x)$ approaches L as x **increases** to c (approaches c from the left).

We write this as

$$\boxed{\lim_{x \to c^-} f(x) = L.}$$

We say that L is the **limit of f as x approaches c from the right** if the value of $f(x)$ approaches R as x **decreases** to c (approaches c from the right).

We write this as

$$\boxed{\lim_{x \to c^+} f(x) = L.}$$

Use a tabl[redacted]
limits: [redacted]

(1) $\lim_{x\to 7^-} 2x+5$ [redacted]

x	6	6.5	6.75	6.9	6.99	6.999
$f(x)$	17	18	18.5	18.8	18.98	18.998

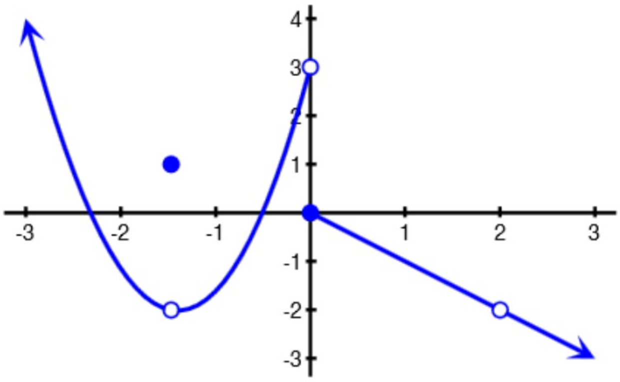
$\frac{\cancel{(x+1)}(x-1)}{(x-2)\cancel{(x+1)}} = \frac{-2}{-3}$

(2) $\lim_{x\to -1^+} \frac{x^2-1}{x^2-x-2} =$ [redacted]

x	0	-0.5	-0.75	-0.9	-0.99	-0.999
$f(x)$	0.5	0.6	$0.\overline{63}$	$0.655\ldots$	$0.6655\ldots$	$0.66655\ldots$

[redacted]

Below is the graph of a function $f(x)$:



Use this graph to determine the following one-sided limits:

(1) $\lim_{x\to -1.5^-} f(x)$ -2 (2) $\lim_{x\to 0^-} f(x)$ 3 (3) $\lim_{x\to 2^-} f(x)$ -2

(4) $\lim_{x\to -1.5^+} f(x)$ -2 (5) $\lim_{x\to 0^+} f(x)$ 0 (6) $\lim_{x\to 2^+} f(x)$ -2

$\lim_{x\to -1.5} = -2$ $\lim_{x\to 0}$ DNE $\lim_{x\to 2}$ -2

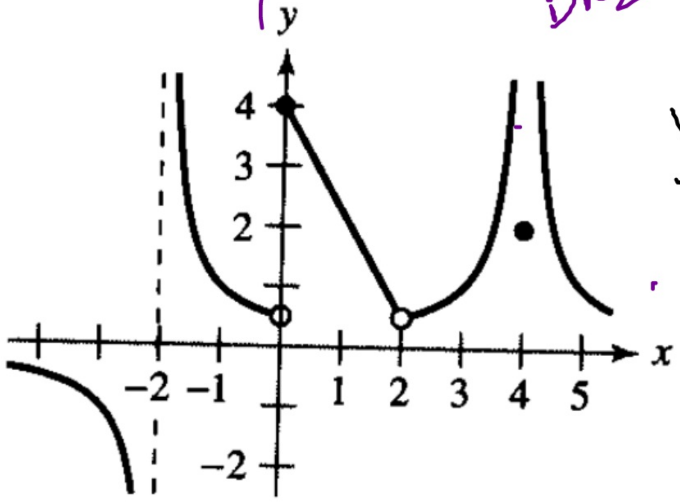
EXAMPLE: Use the graph of f(x) below to find the following:

$f(0)$, $f(2)$, $f(-2)$, $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2} f(x)$,

4 UND UND 1/2 .5 1/2

$\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x)$, $\lim_{x\to -2} f(x)$

1/2 4 DNE DNE



$\lim_{x\to 4^-} = \infty$

$\lim_{x\to 4^+} = \infty$

$\lim_{x\to 4}$ DNE

b.) $f(2) = \text{undef.}$ For this one, there is no closed circle at the x = 2. So, nothing is defined here.

c.) $f(-2) = \text{undef.}$ There is no closed circles here either. We have a vertical asymptote, so nothing will be defined here.

d.) $\lim_{x\to 2^+} f(x) = \frac{1}{2}$ You are seeing what the y-value is approaching as x approaches 2 from the right.

e.) $\lim_{x\to 2^-} f(x) = \frac{1}{2}$ You are seeing what the y-value is approaching as x approaches 2 from the left.

f.) $\lim_{x\to 2} f(x) = \frac{1}{2}$ Since the limit from the left and right are the same then our limit exists and is also equal to 1.

g.) $\lim_{x\to 0^+} f(x) = 4$ You are seeing what the y-value is approaching as x approaches 0 from the right.

h.) $\lim_{x\to 0^-} f(x) = \frac{1}{2}$ You are seeing what the y-value is approaching as x approaches 0 from the left.

i.) $\lim_{x\to 0} f(x) =$ d.n.e. Since the limit from the left and from the right are not the same, the limit does not exist.

g.) $\lim_{x\to -2} f(x) =$ d.n.e. Since the limit from the left and from the right are not the same, the limit does not exist.

Limits at infinity

Evaluate each limit.

5) $\lim_{x \to -\infty} \left(x^3 - 4x^2 + 5\right)$

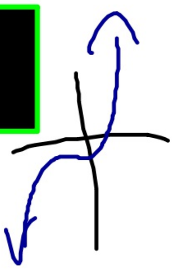
$-\infty$

[redacted]

6) $\lim_{x \to \infty} \frac{2x^3}{3x^2 - 4}$

∞

[redacted]



7) $\lim_{x \to \infty} \frac{x^3}{4x^2 + 3}$

∞

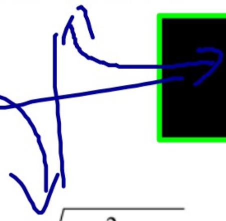
[redacted]

$\frac{\sqrt{2x^2}}{2x}$

8) $\lim_{x \to \infty} \frac{x + 1}{2x^2 + 2x + 1}$

0

$\frac{\sqrt{2}\,x}{2x}$



[redacted]

9) $\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 3}}{2x + 3}$

$-\frac{\sqrt{2}}{2}$

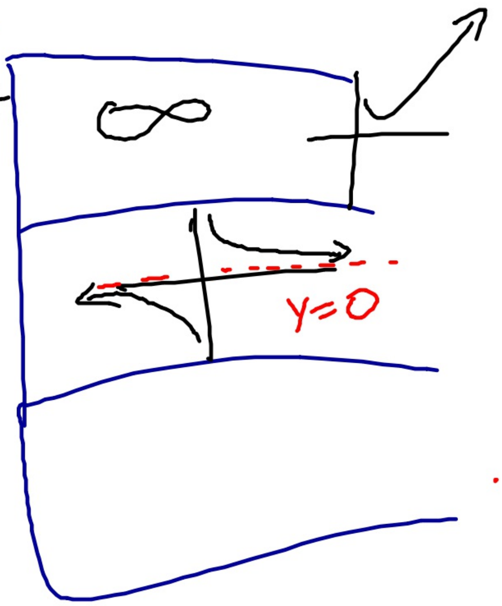
[redacted]

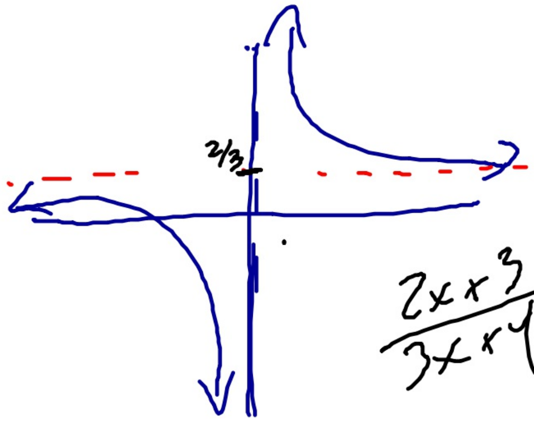
10) $\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 1}}{4x + 2}$

$-\frac{\sqrt{2}}{4}$

[redacted]

DEGREE ON TOP BIGGER





$\frac{2x+3}{3x+4}$