Warm-up

1) $f(x) = 2x^2 + 2$ $g(x) = 2x + 3$ Find $(f - g)(-1)$

2) If $f(x) = 5x^2 - 3$ find $f^{-1}(x)$

3) What is the domain: $f(x) = \frac{\sqrt{x+5}}{x-2}$

4) For $f(x) = x^2$

a) Describe the steps (in order) to graph

$g(x) = -2f(x - 1) + 4$

b) What is the expanded form of $g(x)$?

17. $A = ks^2$

18. $V = kr^2$

19. $I = V/R$

20. $V = kT$

21. $E = mc^2$

22. $p = \sqrt{2gd}$.

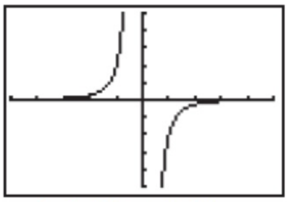
In Exercises 27–30, state the power and constant of variation for the function, graph it, and analyze it in the manner of Example 2 of this section.

30. $f(x) = -2x^{-3}$

30. power $= -3$, constant $= -2$; Domain: $(-\infty, 0) \cup (0, \infty)$;

Range: $(-\infty, 0) \cup (0, \infty)$; Discontinuous at $x = 0$; Increasing on $(-\infty, 0)$ and on $(0, \infty)$.; Odd. Symmetric with respect to origin; Not bounded above or below; No local extrema; Asymptotes at $x = 0$ and $y = 0$.;

End Behavior: $\lim_{x \to -\infty} -2x^{-3} = 0$, $\lim_{x \to \infty} -2x^{-3} = 0$.



$[-5, 5]$ by $[-5, 5]$

51. $V = \frac{kT}{P}$, so $k = \frac{PV}{T} = \frac{(0.926 \text{ atm})(3.46 \text{ L})}{302°\text{K}}$

$= 0.0106 \frac{\text{atm–L}}{\text{K}}$

At $P = 1.452$ atm, $V = \frac{\left(\frac{0.0106 \text{ atm–L}}{\text{K}}\right)(302°\text{K})}{1.452 \text{ atm}}$

$= 2.21$ L

52. $V = kPT$, so $k = \frac{V}{PT} = \frac{(3.46 \text{ L})}{(0.926 \text{ atm})(302°\text{K})}$

$= 0.0124 \frac{\text{L}}{\text{atm–K}}$

At $T = 338°$K, $V = \left(0.0124 \frac{\text{L}}{\text{atm–K}}\right)(0.926 \text{ atm})$

$(338°\text{K}) = 3.87$ L

55. Keeping Warm For mammals and other warm-blooded animals to stay warm requires quite a bit of energy. Temperature loss is related to surface area, which is related to body weight, and temperature gain is related to circulation, which is related to pulse rate. In the final analysis, scientists have concluded that the pulse rate r of mammals is a power function of their body weight w.

(b) Find the power regression model. [redacted]

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression model to predict the pulse rate for a 450-kg horse. Is the result close to the 38 beats/min reported by A. J. Clark in 1927?

Table 2.12 Weight and Pulse Rate of Selected Mammals

Mammal	Body weight (kg)	Pulse rate (beats/min)
Rat	0.2	420
Guinea pig	0.3	300
Rabbit	2	205
Small dog	5	120
Large dog	30	85
Sheep	50	70
Human	70	72

Source: A. J. Clark, Comparative Physiology of the Heart. New York: Macmillan, 1927.

57. Light Intensity Velma and Reggie gathered the data in Table 2.13 using a 100-watt light bulb and a Calculator-Based Laboratory™ (CBL™) with a light-intensity probe.

(a) Draw a scatter plot of the data in Table 2.13

(b) Find the power regression model. Is the power close to the theoretical value of $a = -2$? [redacted]

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression model to predict the light intensity at distances of 1.7m and 3.4 m.

Table 2.13 Light Intensity Data for a 100-W Light Bulb

Distance (m)	Intensity (W/m^2)
1.0	7.95
1.5	3.53
2.0	2.01
2.5	1.27
3.0	0.90

Objective: Describe and graph polynomial equations of higher degrees

1. What is standard form of a polynomial equation?

AS IN DOG.

$Ax^3 + Bx^2 + Cx + D = 0$

~~$5x^2 + 2x^3 - 4x$~~

2. How do you determine the degree of a polynomial equation?

LARGEST EXPONENT

3. What is the leading coefficent of a polynomial equation?

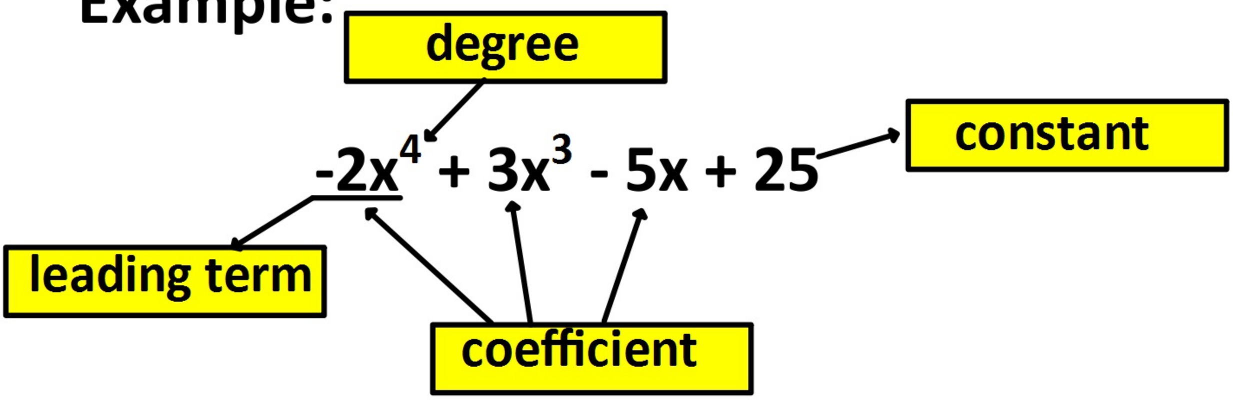
FIRST COEF. IN STANDARD FORM

4. What degree is a quadratic equation? 2ND.

5. What degree is a cubic equation? 3RD

6. What degree is a quartric equation? 4TH

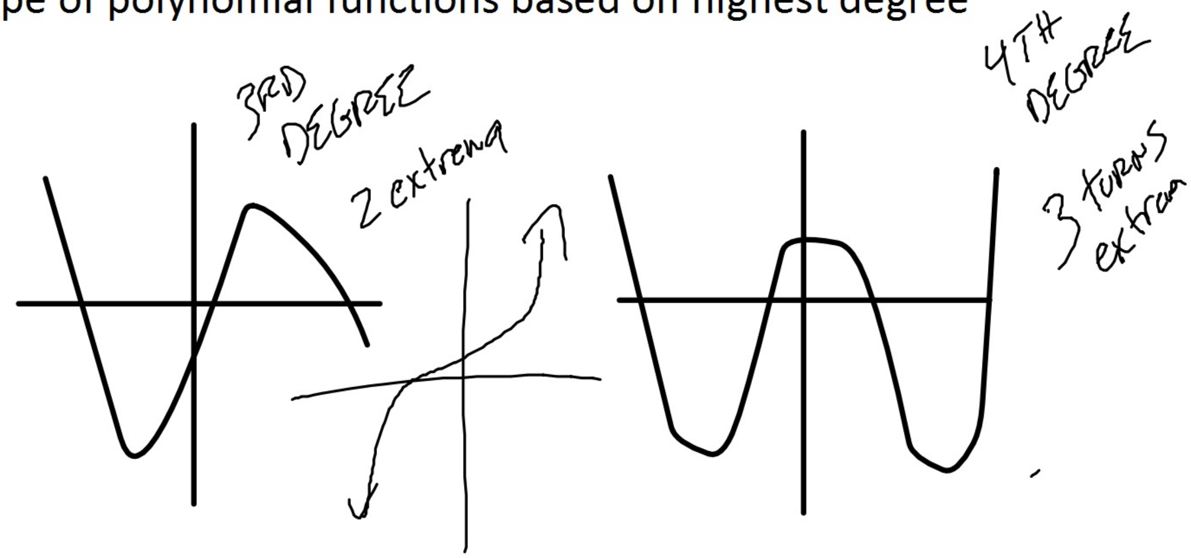
Example:



Classify the polynomial by number of terms and by degree?

1. $f(x) = x^3 + 4$ 3RD DEGREE BINOMIAL
2. $f(x) = 2x^2 + 7x - 5$ 2ND DEGREE TRINOMIAL
3. $f(x) = 4x^4 + 5x - 3$ 4TH " " " "
4. $f(x) = 3x$ FIRST DEGREE MONOMIAL

Shape of polynomial functions based on highest degree



Local extrema

Zeros = DEGREE

A polynomial functions of degree n has at most n - 1 local extrema and at most n zeros.

End Behavior of Polynomial Functions

An important characteristic of polynomial functions is their **end behavior**. As we shall see, the end behavior of a polynomial is closely related to the **end behavior of its leading term**. Exploration 1 examines the end behavior of monomial functions, which are potential leading terms for polynomial functions.

With a partner:

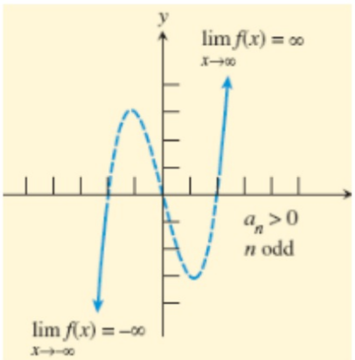
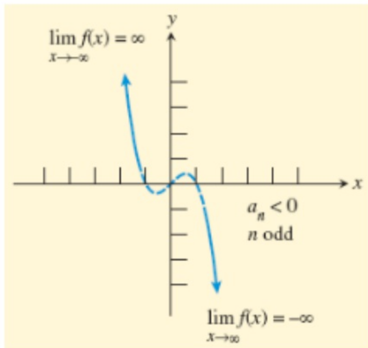
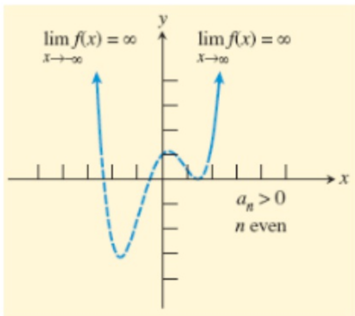
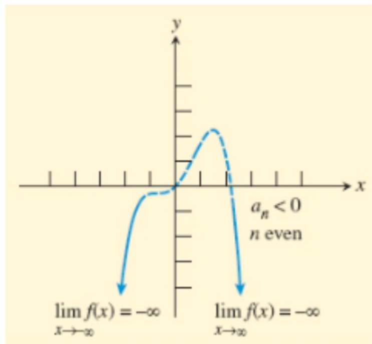
EXPLORATION 1 Investigating the End Behavior of $f(x) = a_n x^n$

Graph each function in the window $[-5, 5]$ by $[-15, 15]$. Describe the end behavior using $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

Describe the patterns you observe. In particular, how do the values of the coefficient a_n and the degree n affect the end behavior of $f(x) = a_n x^n$?

1. (a) $f(x) = 2x^3$ (b) $f(x) = -x^3$
   (c) $f(x) = x^5$ (d) $f(x) = -0.5x^7$
2. (a) $f(x) = -3x^4$ (b) $f(x) = 0.6x^4$
   (c) $f(x) = 2x^6$ (d) $f(x) = -0.5x^2$
3. (a) $f(x) = -0.3x^5$ (b) $f(x) = -2x^2$
   (c) $f(x) = 3x^4$ (d) $f(x) = 2.5x^3$

Leading Term Test for End Behavior

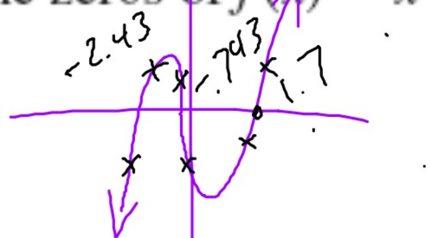
	leading coefficient is positive	leading coefficient is negative
odd degree		
even degree		

Zeros of Polynomial Functions

Recall that finding the real-number zeros of a function f is equivalent to finding the x-intercepts of the graph of $y = f(x)$ or the solutions to the equation $f(x) = 0$. Example 5 illustrates that *factoring* a polynomial function makes solving these three related problems an easy matter.

Example 4:

Find the zeros of $f(x) = x^3 - x^2 - 6x$.



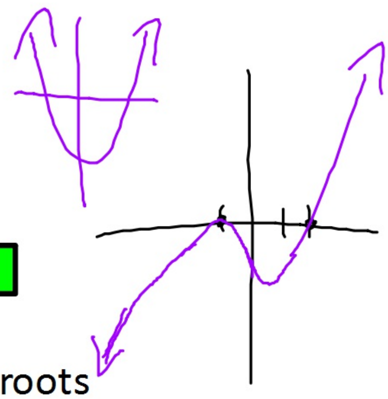
Approximate the zeros: $2x^3 + 3x^2 - 7x - 6$

$x^4 + 3x^3 - 9x^2 + 2x + 3$

What if it's in factored form???

4TH 5TH DEGREE

like $f(x) = (x - 4)^4$ or $g(x) = (x - 2)^3(x + 1)^2$



How many roots should the have?

f(x) should have 4 roots g(x) should have 5 roots

DEFINITION Multiplicity of a Zero of a Polynomial Function

If f is a polynomial function and $(x - c)^m$ is a factor of f but $(x - c)^{m+1}$ is not, then c is a zero of **multiplicity** m of f.

In other words, if the multiplicity is even the zero is a "touch and go"

but if the multiplicity is odd, the graph will cut through the zero!

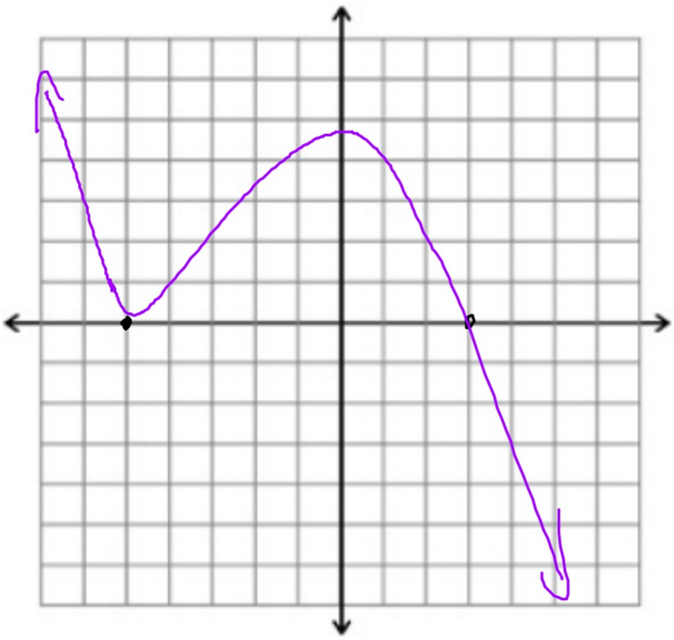
State the degree and list the zeros of the function $f(x)$ **BELOW** State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of f by hand.

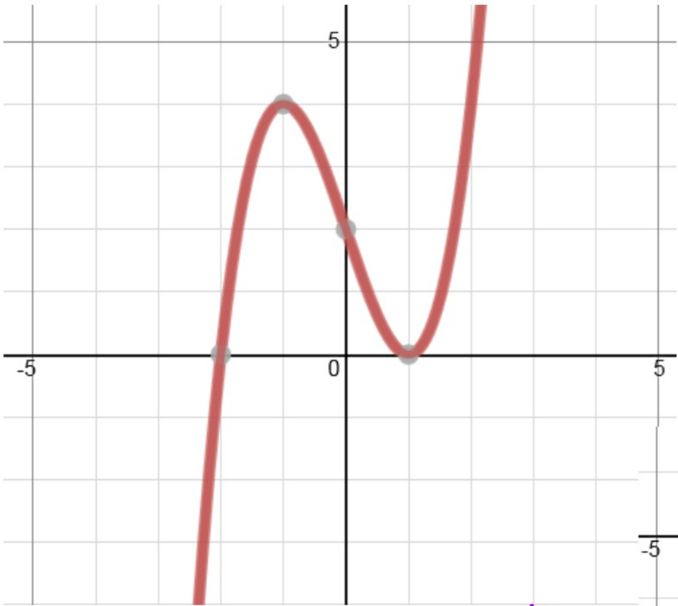
1) $(x - 2)^3 (x + 1)^2$

2) $- (x + 3)^4 (x - 4)^2$

3) $x^2 - 9$

4) $- (x - 3)^3 (x + 5)^2$





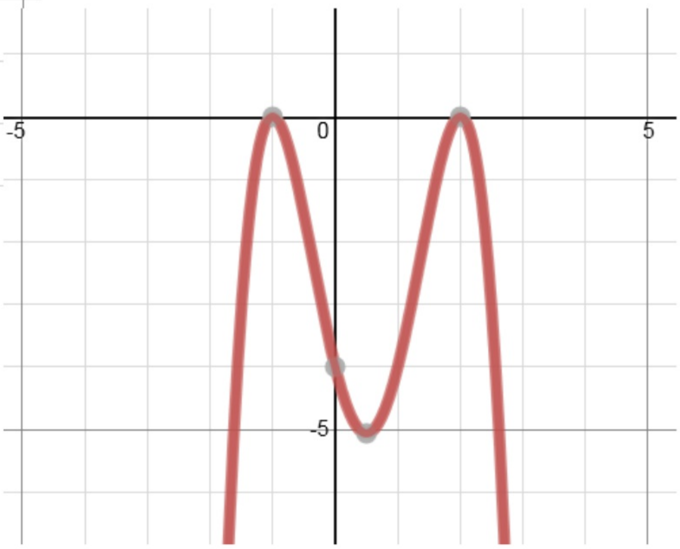
Find the function:

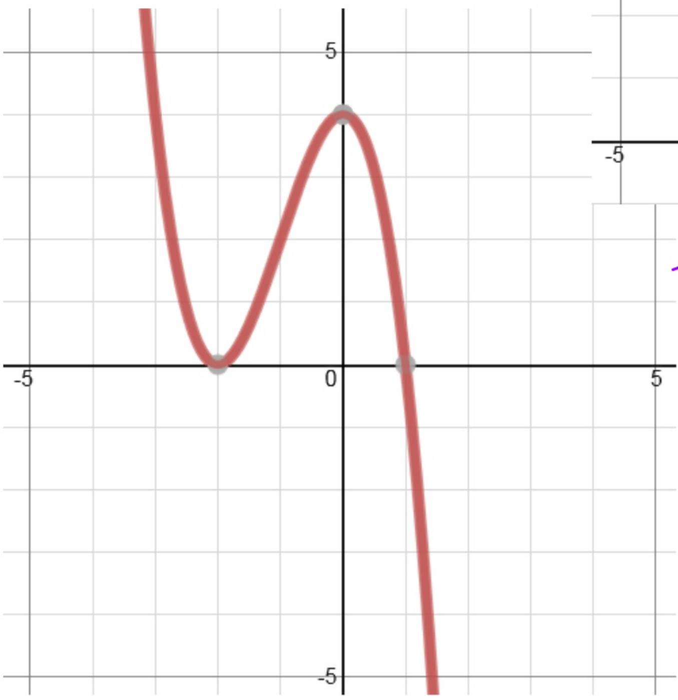
(factored form)

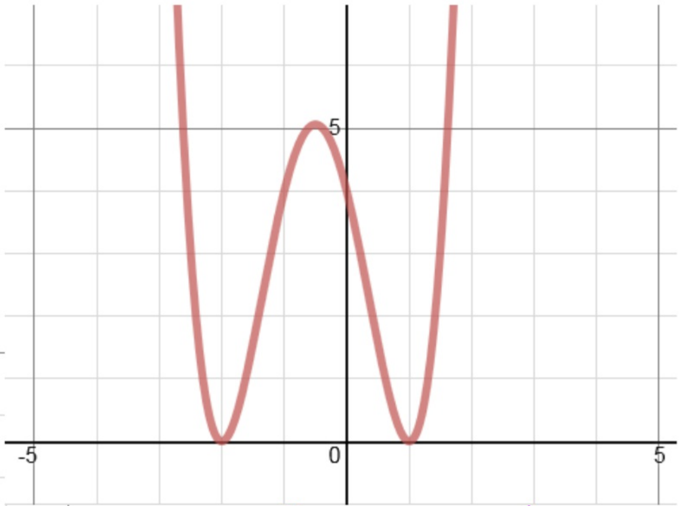
minimum

$-(x+1)^2(x-2)^2$

$(x+2)^1(x-1)^2$







$-(x+2)^2(x-1)^1$

Find the cubic function with the given zeros:

1) 3, -4, 6

2) $\sqrt{3}$, $-\sqrt{3}$, 4

$x+\sqrt{3}$ $x-\sqrt{3}$

$(x^2-3)(x-4)$

$x^3-4x^2-3x+12$