Warm up

1) Describe the end behavior of the function $-3x^3 - 4x^2 + 2$ using limit notation

2) Sketch the graph: $(x - 2)^3 (x + 1)^2$

3) The frequency of a vibrating string varies inversely as its length. A string 3 feet long vibrates 175 cycles per second. Find the frequency of a 5 foot string.

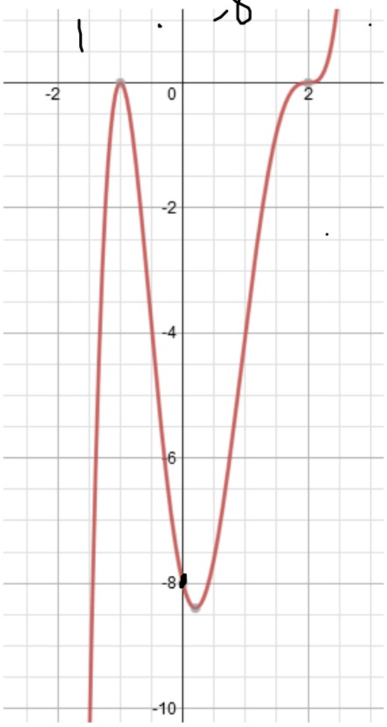
4) The cost of pipe at Lowe's varies jointly as the length and diameter of the pipe. If 20 feet of 0.5 inch diameter pipe costs $36.60, then what is the cost of 12 feet of 0.75 inch diameter pipe?

Warm up

1)

2)

$$(\overset{0}{x}+1)^2 \cdot (\overset{0}{x}-2)^3 = 0$$

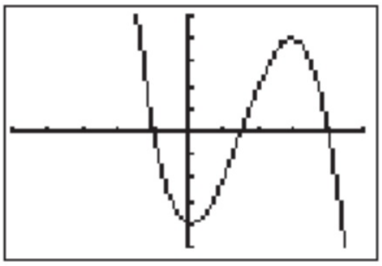


3) 105 cycles/sec.

4) $32.94

8. local maximum: $(0, 0)$, local minima: $\approx (1.12, -3.13)$ and $(-1.12, -3.13)$, zeros: $x = 0$, $x \approx 1.58$, $x \approx -1.58$.

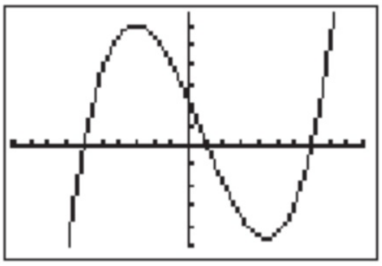
18.



$[-5, 5]$ by $[-15, 15]$

$\lim_{x \to \infty} f(x) = -\infty$; $\lim_{x \to -\infty} f(x) = \infty$

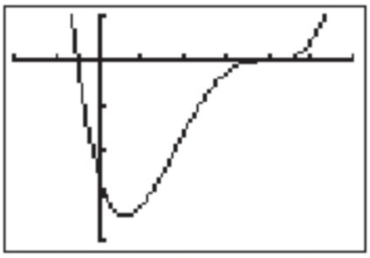
20.



$[-10, 10]$ by $[-100, 130]$

$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = -\infty$

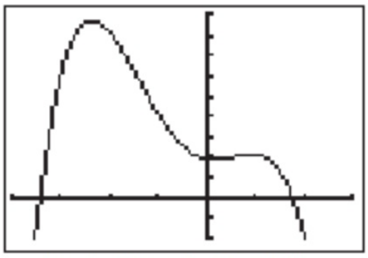
22.



$[-2, 6]$ by $[-100, 25]$

$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = \infty$

24.



$[-4, 3]$ by $[-20, 90]$

$\lim_{x \to \infty} f(x) = -\infty$; $\lim_{x \to -\infty} f(x) = -\infty$

26. $f(x) = -x^3 + 7x^2 - 4x + 3$ $-\infty, \infty$

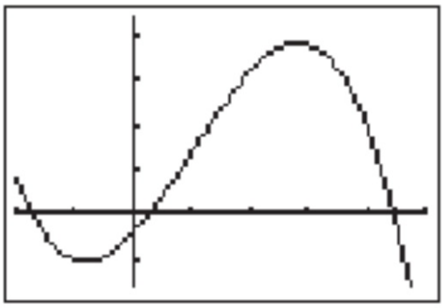
34. $f(x) = 3x^2 + 4x - 4$ -2 and $\frac{2}{3}$

36. $f(x) = x^3 - 25x$ 0, -5, and 5

38. $f(x) = 5x^3 - 5x^2 - 10x$

0, -1, and 2

44.



$[-2, 5]$ by $[-8, 22]$

-1.73, 0.26, 4.47

52. $f(x) = x^3 - 4x^2 - 44x + 96$ -6, 2, and 8

54. $f(x) = (x + 2)(x - 3)(x + 5) = x^3 + 4x^2 - 11x - 30$

73) B

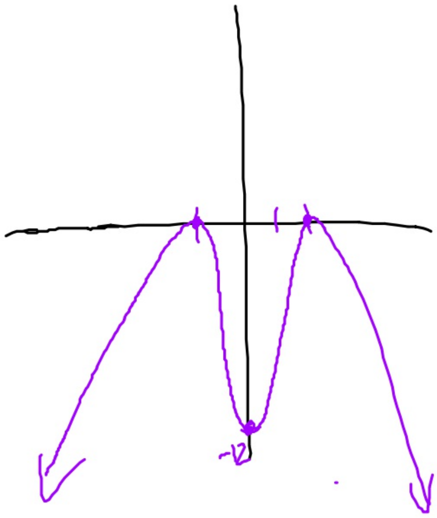
74) A

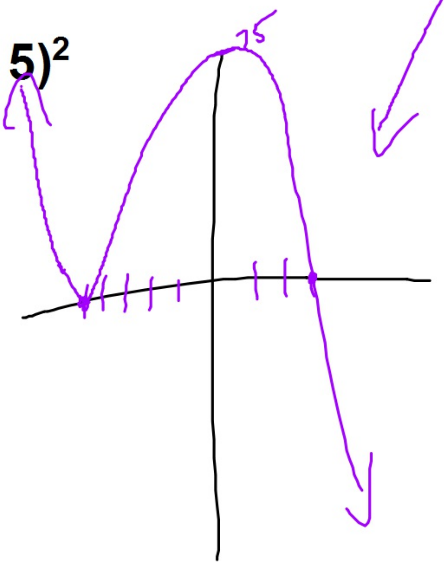
Graphing in factored form...

1) $(x - 2)^3 (x + 1)^2$

2) $-3(x + 1)^2 (x - 2)^2$

3) $-(x - 3)(x + 5)^2$





Objective: Find Zeros by dividing polynomials
Find the remainder of a root
Determine whether a binomial is a factor of a polynomial

Divide using long divison:

$$\frac{3x^3 + 5x^2 + 8x + 7}{3x + 2}$$

$$x^2 + x + 2 + \frac{3}{3x+2}$$

$$3x+2 \,\overline{\big)\, 3x^3 + 5x^2 + 8x + 7}$$

$$- \quad \cancel{3}x^3 + 2x^2$$

$$3\cancel{x}^2 + 8x$$

$$- \quad \cancel{3}x^2 + 2x$$

$$6x + 7$$

$$\underline{6x + 4}$$

$$3$$

Using Polynomial Long Division

…e quotient and remainder when $2x^4 - x^3 - 2$ is divided by $2x^2 + x + 1$.

$$\left(x^2 - x + \frac{x-2}{2x^2+x+1}\right)$$

$$(2x^2+x+1)\overline{\big)\, 2x^4 - x^3 + 0x^2 + 0x - 2}$$

$$-\;\; \cancel{2x^4} + x^3 + x^2$$

$$-\cancel{2x^3} - \cancel{x^2} + 0x$$

$$-\;\; -\cancel{2x^3} - \cancel{x^2} - x$$

$$x^1 - 2$$

Using Synthetic Division

[redacted]e a summary

Divide Synthetically: $\dfrac{(x^4 - 3x^3 + 5x - 6)\cancel{(x+2)^{-1}}}{(x+2)}$

$$\begin{array}{r|rrrrr} & 1 & -3 & 0 & 5 & -6 \\ -2 & & -2 & 10 & -20 & 30 \\ \hline & 1 & -5 & 10 & -15 & \boxed{24} \end{array}$$

$$(x+2)\left(x^3-5x^2+10x-15\right)+\frac{24}{x+2}$$

THEOREM Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

EXAMPLE 2 Using the Remainder Theorem

Find the remainder when $f(x) = 3x^2 + 7x - 20$ is divided by $x + 4$.

$f(-4)$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ & & -12 & 20 \\ \hline & 3 & -5 & 0 \end{array}$$

THEOREM Factor Theorem

A polynomial function $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

In Exercises 19–24, use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

$x - 2$; $x^3 - 3x - 2$ $\quad (2)^3 - 3(2) - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$(x^2+2x+1)(x-2)$

$(x+1)^2$

Fundamental Connections for Polynomial Functions

For a polynomial function f and a real number k, the following statements are equivalent:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$.
2. k is a zero of the function f.
3. k is an x-intercept of the graph of $y = f(x)$.
4. $(x - k)$ is a factor of $f(x)$.

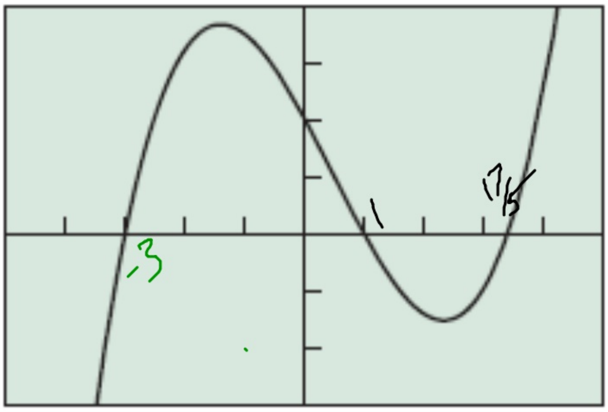
In Exercises 25 and 26, use the graph to guess possible linear factors of $f(x)$. Then completely factor $f(x)$ with the aid of synthetic division.

$f(x) = 5x^3 - 7x^2 - 49x + 51$

$$\begin{array}{r|rrrr} -3 & 5 & -7 & -49 & 51 \\ & & -15 & 66 & -51 \\ \hline & 5 & -22 & 17 & 0 \end{array}$$

$5x^2 - 22x + 17$

$(5x-17)(x-1)$



[–5, 5] by [–75, 100]

THEOREM Rational Zeros Theorem

Suppose f is a polynomial function of degree $n \geq 1$ of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0,$$

with every coefficient an integer and $a_0 \neq 0$. If $x = p/q$ is a rational zero of f, where p and q have no common integer factors other than 1, then

- p is an integer factor of the constant coefficient a_0, and
- q is an integer factor of the leading coefficient a_n.

$$\frac{p}{q} \quad \frac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 3}$$

$\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{2}{1}, \pm\frac{2}{3},$

Finding the Rational Zeros

In Exercises 33–36, use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros.

$f(x) = 3x^3 - 7x^2 + 6x - 14$

$\pm\frac{7}{1}, \pm\frac{7}{3}, \pm\frac{14}{3}, \pm\frac{14}{1}$

Finding the Real Zeros of a Polynomial Function

Find all of the real zeros of $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$.

Rational zeros:

$\frac{\pm 1, 2, 4, 8}{\pm 1, 2}$

$\pm 1,$

Classwork

In Exercises 1–6, divide $f(x)$ by $d(x)$, and write a summary statement in polynomial form and fraction form.

1. $f(x) = x^2 - 2x + 3$; $d(x) = x - 1$
2. $f(x) = x^3 - 1$; $d(x) = x + 1$
3. $f(x) = x^3 + 4x^2 + 7x - 9$; $d(x) = x + 3$
4. $f(x) = 4x^3 - 8x^2 + 2x - 1$; $d(x) = 2x + 1$