Warm up

Simplify the foll

1. $\frac{n+2!}{n!}$

2) $\frac{n-1!}{n+1!}$

Find the recursive for 17) $a_n = a_{n-1} - 30$, $a_1 = -28$

17) $-28, -58, -88, -118, \ldots$

18) $a_n = a_{n-1} + 20$, $a_1 = -3$

$-3, 17, 37, 57, \ldots$

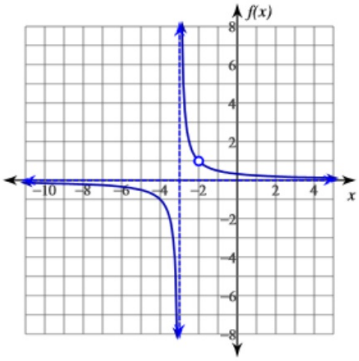
Find the recursive forr 25) $a_n = a_{n-1} \cdot -5$, $a_1 = 2$

25) $2, -10, 50, -250, \ldots$

26) $a_n = a_{n-1} \cdot 3$, $a_1 = 2$

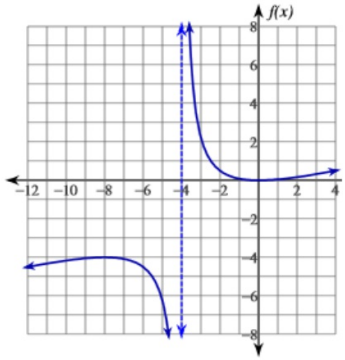
$2, 6, 18, 54, \ldots$

$x \to -3^+$ $x^2 + 5x + 6$



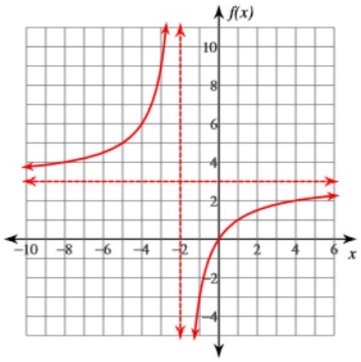
∞

$x \to -4$ $4x + 16$



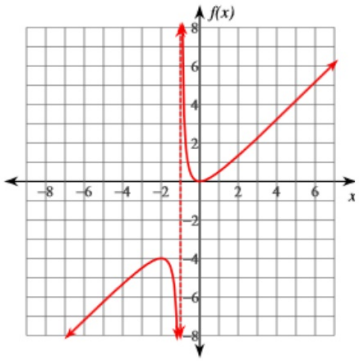
Does not exist.

3) $\lim_{x \to -2^+} \frac{3x}{x+2}$



$-\infty$

4) $\lim_{x \to -1^+} \frac{x^2}{x+1}$



∞

5) $\lim_{x\to -3^-} \frac{2x}{x+3}$

∞

6) $\lim_{x\to -2^+} \frac{1}{x^2-4}$

$-\infty$

9) $\lim_{x\to -2^-} \frac{x+2}{x^2+x-2}$

$-\frac{1}{3}$

10) $\lim_{x\to -3^-} -\frac{2}{x+3}$

∞

-3.002	-3.001	-3	
3.002	6.002		

7) $\lim_{x\to 3^-} -\frac{4x}{x-3}$

∞

8) $\lim_{x\to 1} -\frac{3}{x-1}$

Does not exist.

11) $\lim_{x\to \frac{\pi}{4}^-} 2\sec(2x)$

∞

12) $\lim_{x\to \frac{3\pi}{4}^+} 2\tan(2x)$

$-\infty$

Sequence, Series, and Limits Test
Unit Objectives

Sequences

- Write out the terms to a sequence defined recursively.
- Determine whether a sequence converges or diverges. If a sequence converges, evaluate what the sequence converges to.
- Write the n^{th} term explicit equation for arithmetic and geometric sequences.
- Write the explicit equation given terms in a sequence (example, given a_3 and a_{11})
- Write a recursive rule for arithmetic and geometric sequences.

Series

- Write a series using sigma notation.
- Evaluate arithmetic and geometric series.
- Determine whether a series converges or diverges. If a series converges, evaluate what the series converges to.
- Solve applications of geometric series.

Limits

- Evaluate limits using algebraic methods.
  - Direct Substitution, Factoring, Fractions, Rationalizing.
- Evaluate limits using a graph.
- Find one-sided limits.
- Determine whether a limit exists using one-sided limits.
- Evaluate limits going to +/- infinity.

Other

- Simplify factorial expressions.

Last person in each row
Grab a white board...

Find the limit, if one exists...

1) $\lim_{x\to\infty} \dfrac{-5x^3 - 2x + 4}{x^4}$

2) $\lim_{x\to-\infty} \dfrac{1}{x}$

3) $\lim_{x\to\infty} \dfrac{-5x^3 - 2x + 4}{x^3}$

4) $\lim_{x\to\infty} \dfrac{x^2 + 9}{5x + 2x^2 - 3}$

5) $\lim_{x\to-\infty} \dfrac{x + x^2 - 6}{6x + 5x^2 + 2x^3}$

6) $\lim_{x\to-\infty} \dfrac{3x - 2}{x}$

Find the one-sided limits below for the graph f(x)

1) $\lim_{x\to -1^-} f(x)$

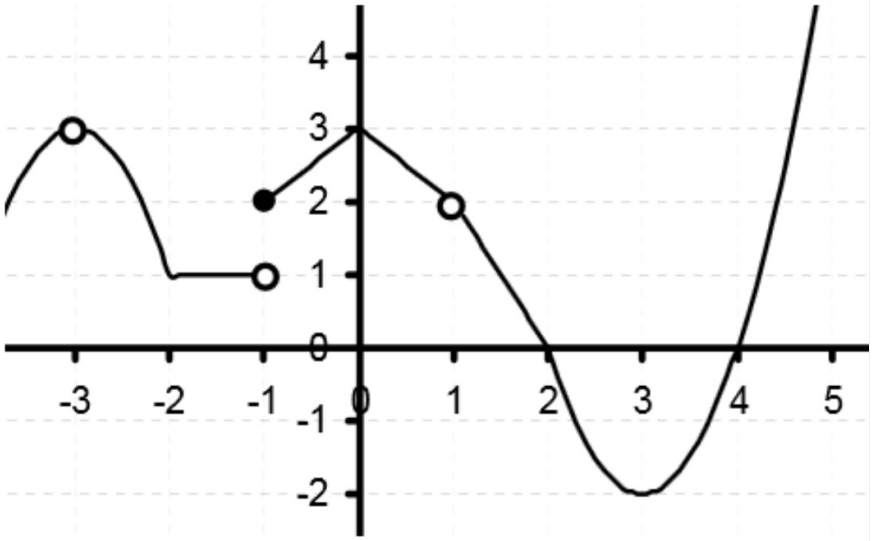
2) $\lim_{x\to 3^+} f(x)$

3) $\lim_{x\to -3^+} f(x)$

4) $\lim_{x\to 2^-} f(x)$

5) $\lim_{x\to 0^-} f(x)$

6) $\lim_{x\to -1^+} f(x)$



Find the limit, if one exists...

1) $\lim_{x\to 3} \dfrac{x^2-9}{x^2+2x-15}$

2) $\lim_{x\to -2} \dfrac{x^2-4}{x+2}$

3) $\lim_{x\to -3} \dfrac{x^2+7x+12}{x^2-9}$

4) $\lim_{x\to 3} \dfrac{x^2-x-6}{x-3}$

5) $\lim_{x\to 2} \dfrac{x^2-x-2}{x^2-4}$

6) $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-3x+2}$

Find the limits below for the graph f(x)

1) $\lim_{x\to -3} f(x)$

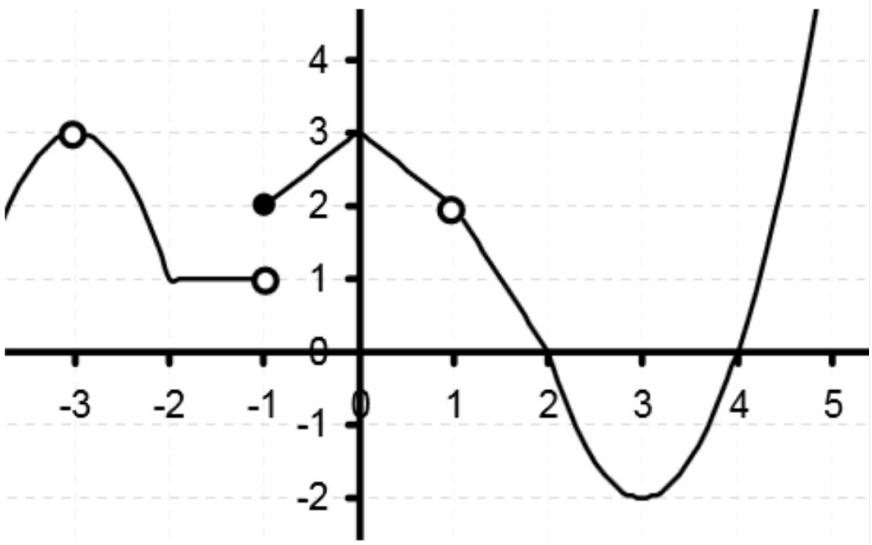
2) $\lim_{x\to -2} f(x)$

3) $\lim_{x\to -1} f(x)$

4) $\lim_{x\to 1} f(x)$

5) $\lim_{x\to 2} f(x)$

6) $\lim_{x\to 3} f(x)$



Find the limits below as f(x) approaches - ∞

1) $f(x) = -x^3 + 2x^2 + 2$

2) $f(x) = -x^4 + x^2 + 2$

3) $f(x) = x^3 - 3x^2 + 1$

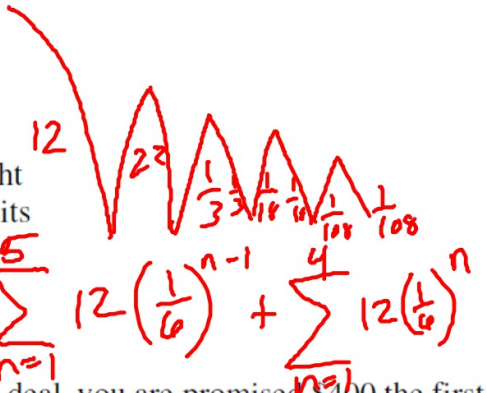
4) $f(x) = x^3 + 10x^2 + 32x + 34$

5) $f(x) = -x^5 + 4x^3 - 5x - 4$

6) $f(x) = x^4 - 3x^2 - 3x + 4$

Word Problems

1) A rubber ball dropped on a hard surface takes a sequence of bounces, each one $\frac{1}{6}$ as high as the preceding one. If this ball is dropped from a height of 12 feet, how far will it have traveled when it hits the surface the fifth time?



$$\sum_{n=1}^{5} 12\left(\frac{1}{6}\right)^{n-1} + \sum_{n=1}^{4} 12\left(\frac{1}{6}\right)^{n}$$

a. $14\frac{43}{108}$ ft

b. $16\frac{217}{270}$ ft

c. $28\frac{43}{54}$ ft

d. $16\frac{43}{54}$ ft

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

2) In a financial deal, you are promised \$400 the first day and each day after that you will receive 65% of the previous day's amount. When one day's amount drops below \$1, you stop getting paid from that day on. What day is the first day you would receive no payment and what is your total income?

$a_n = a_1 \cdot r^{n-1}$

a. 13th day; \$1142.03 total income

b. 15th day; \$1140.11 total income

c. 15th day; \$1139.15 total income

d. 21st day; \$1140.11 total income

1141.

Properties of limits

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Use properties of limits to evaluate

1. $\lim_{x \to c} f(x) = 3, \quad \lim_{x \to c} g(x) = 6$

(a) $\lim_{x \to c} [-2g(x)]$ (b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} \frac{f(x)}{g(x)}$ (d) $\lim_{x \to c} \sqrt{f(x)}$

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2. $\lim_{x \to c} f(x) = 5, \quad \lim_{x \to c} g(x) = -2$

(a) $\lim_{x \to c} [f(x) + g(x)]^2$ (b) $\lim_{x \to c} [6f(x)g(x)]$

(c) $\lim_{x \to c} \frac{5g(x)}{4f(x)}$ (d) $\lim_{x \to c} \frac{1}{\sqrt{f(x)}}$