Warm up

Simplify the following:
1. \( \frac{n+2}{n!} \)
2. \( \frac{n-1}{n+1!} \)

Find the recursive formula:
17) \( a_n = a_{n-1} - 30 \)  
17) \( a_1 = -28 \)

18) \( a_n = a_{n-1} + 20 \)  
18) \( a_1 = -3 \)

Find the recursive formula:
25) \( a_n = a_{n-1} \cdot -5 \)  
25) \( a_1 = 2 \)

26) \( a_n = a_{n-1} \cdot 3n \)  
26) \( a_1 = 2 \)

20) \( 2, 6, 18, 54, ... \)
5) \( \lim_{x \to -3} \frac{2x}{x + 3} \)  
6) \( \lim_{x \to -2} \frac{1}{x^2 - 4} \)  
9) \( \lim_{x \to 2} \frac{x + 2}{x^2 + x - 2} \)  
10) \( \lim_{x \to -3} \frac{-2}{x + 3} \)  

\(-\infty\) \(-\infty\) \(-1/3\) \(-\infty\)

7) \( \lim_{x \to 3} \frac{-4x}{x - 3} \)  
8) \( \lim_{x \to 1} \frac{3}{x - 1} \)  
11) \( \lim_{x \to \pi/4} 2 \sec (2x) \)  
12) \( \lim_{x \to 3\pi/4} 2 \tan (2x) \)  

Does not exist. 

\(\tan \frac{3\pi}{2}\) 

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**Sequence, Series, and Limits Test**

**Unit Objectives**

**Sequences**
- Write out the terms to a sequence defined recursively.
- Determine whether a sequence converges or diverges. If a sequence converges, evaluate what the sequence converges to.
- Write the \(n^{th}\) term explicit equation for arithmetic and geometric sequences.
- Write the explicit equation given terms in a sequence (example, given \(a_3\) and \(a_{11}\))
- Write a recursive rule for arithmetic and geometric sequences.

**Series**
- Write a series using sigma notation.
- Evaluate arithmetic and geometric series.
- Determine whether a series converges or diverges. If a series converges, evaluate what the series converges to.
- Solve applications of geometric series.

**Limits**
- Evaluate limits using algebraic methods.
  - Direct Substitution, Factoring, Fractions, Rationalizing.
- Evaluate limits using a graph.
- Find one-sided limits.
- Determine whether a limit exists using one-sided limits.
- Evaluate limits going to +/- infinity.

**Other**
- Simplify factorial expressions.
Last person in each row
Grab a white board...

Find the limit, if one exists...

1) \( \lim_{{x \to \infty}} \frac{-5x^3 - 2x + 4}{x^4} \)

2) \( \lim_{{x \to -\infty}} \frac{1}{x} \)

3) \( \lim_{{x \to \infty}} \frac{-5x^3 - 2x + 4}{x^3} \)

4) \( \lim_{{x \to \infty}} \frac{x^2 + 9}{5x + 2x^2 - 3} \)

5) \( \lim_{{x \to -\infty}} \frac{x + x^2 - 6}{6x + 5x^2 + 2x^3} \)

6) \( \lim_{{x \to -\infty}} \frac{3x - 2}{x} \)
Find the one-sided limits below for the graph $f(x)$

1) $\lim_{x \to -1^-} f(x)$
2) $\lim_{x \to 3^+} f(x)$
3) $\lim_{x \to -3^+} f(x)$
4) $\lim_{x \to 2^-} f(x)$
5) $\lim_{x \to 0^-} f(x)$
6) $\lim_{x \to -1^+} f(x)$

Find the limit, if one exists...

1) $\lim_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 15}$
2) $\lim_{x \to -2} \frac{x^2 - 4}{x + 2}$
3) $\lim_{x \to -3} \frac{x^2 + 7x + 12}{x^2 - 9}$
4) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$
5) $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 4}$
6) $\lim_{x \to -1} \frac{x^2 + x - 2}{x^2 - 3x + 2}$
Find the limits below for the graph $f(x)$

1) $\lim_{x \to -3} f(x)$
2) $\lim_{x \to -2} f(x)$
3) $\lim_{x \to -1} f(x)$
4) $\lim_{x \to 1} f(x)$
5) $\lim_{x \to 2} f(x)$
6) $\lim_{x \to 3} f(x)$

Find the limits below as $f(x)$ approaches $-\infty$

1) $f(x) = -x^3 + 2x^2 + 2$
2) $f(x) = -x^4 + x^2 + 2$
3) $f(x) = x^3 - 3x^2 + 1$
4) $f(x) = x^3 + 10x^2 + 32x + 34$
5) $f(x) = -x^5 + 4x^3 - 5x - 4$
6) $f(x) = x^4 - 3x^2 - 3x + 4$
Word Problems

1) A rubber ball dropped on a hard surface takes a sequence of bounces, each one \( \frac{1}{6} \) as high as the preceding one. If this ball is dropped from a height of 12 feet, how far will it have traveled when it hits the surface the fifth time?

   a. \( \frac{43}{108} \) ft
   b. \( \frac{217}{270} \) ft
   c. \( \frac{43}{54} \) ft
   d. \( \frac{43}{54} \) ft

   In a financial deal, you are promised $400 the first day and each day after that you will receive 65% of the previous day’s amount. When one day’s amount drops below $1, you stop getting paid from that day on. What day is the first day you would receive no payment and what is your total income?

   a. 13th day; $1142.03 total income
   b. 15th day; $1140.11 total income
   c. 15th day; $1139.15 total income
   d. 21st day; $1140.11 total income

Properties of limits

Properties of Limits
Let \( b \) and \( c \) be real numbers, let \( n \) be a positive integer, and let \( f \) and \( g \) be functions with the following limits.

\[
\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K
\]

1. Scalar multiple:
   \[
   \lim_{x \to c} [b f(x)] = bL
   \]

2. Sum or difference:
   \[
   \lim_{x \to c} [f(x) \pm g(x)] = L \pm K
   \]

3. Product:
   \[
   \lim_{x \to c} [f(x)g(x)] = LK
   \]

4. Quotient:
   \[
   \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}, \quad \text{provided } K \neq 0
   \]

5. Power:
   \[
   \lim_{x \to c} [f(x)]^n = L^n
   \]
Use properties of limits to evaluate

1. \( \lim_{x \to c} f(x) = 3, \quad \lim_{x \to c} g(x) = 6 \)
   
   (a) \( \lim_{x \to c} [-2g(x)] \) \quad (b) \( \lim_{x \to c} [f(x) + g(x)] \)
   
   (c) \( \lim_{x \to c} \frac{f(x)}{g(x)} \) \quad (d) \( \lim_{x \to c} \sqrt{f(x)} \)

2. \( \lim_{x \to c} f(x) = 5, \quad \lim_{x \to c} g(x) = -2 \)
   
   (a) \( \lim_{x \to c} [(f(x) + g(x))^2] \) \quad (b) \( \lim_{x \to c} [6f(x)g(x)] \)
   
   (c) \( \lim_{x \to c} \frac{5g(x)}{4f(x)} \) \quad (d) \( \lim_{x \to c} \frac{1}{\sqrt{f(x)}} \)