

Warm-up

In Exercises 51–56, find all solutions to the equation in the interval $[0, 2\pi)$. You do not need a calculator.

51. $2 \cos x \sin x - \cos x = 0$ $\pi/6, \pi/2, 5\pi/6, 3\pi/2$

52. $\sqrt{2} \tan x \cos x - \tan x = 0$ $0, \pi/4, \pi, 7\pi/4$

54. $\sin x \tan^2 x = \sin x$ $0, \pi/4, 3\pi/4, \pi, 5\pi/4, 7\pi/4$

56. $2 \sin^2 x = 1$ $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

In Exercises 57–62, find all solutions to the equation. You do not need a calculator.

57. $4 \cos^2 x - 4 \cos x + 1 = 0$ 58. $2 \sin^2 x + 3 \sin x + 1 = 0$

59. $\sin^2 \theta - 2 \sin \theta = 0$

60. $3 \sin t = 2 \cos^2 t$

$\pm \frac{\pi}{3} + 2n\pi, n = 0, \pm 1, \pm 2, \dots$ 58. $-\frac{\pi}{6} + 2n\pi, -\frac{5\pi}{6} + 2n\pi, -\frac{\pi}{2} + 2n\pi, n = 0, \pm 1, \pm 2, \dots$

$n\pi, n = 0, \pm 1, \pm 2, \dots$ 60. $\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, n = 0, \pm 1, \pm 2, \dots$ 61. $n\pi, n = 0, \pm 1, \pm 2, \dots$

$\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, n = 0, \pm 1, \pm 2, \dots$ 63. $\{\pm 1.1918 + 2n\pi \mid n = 0, \pm 1, \pm 2, \dots\}$

23 The function $C(x) = \frac{2.50x + 1.00}{x}$ models the cost per item for a company to produce x items after the first item is made. What is the inverse function of $C(x)$?

A $C^{-1}(x) = \frac{1.00}{x - 2.50}$

B $C^{-1}(x) = \frac{x - 2.50}{1.00}$

C $C^{-1}(x) = \frac{x - 1.00}{2.50}$

D $C^{-1}(x) = \frac{2.50}{x - 1.00}$

14 Which is a solution set to $x + \frac{3x}{x-1} = \frac{x+2}{x-1}$?

A $\{-1\}$

B $\{-2\}$

C $\{-2, 1\}$

D $\{2, -1\}$

17 What is the inverse function of $f(x) = \log_5(2x - 1)$?

A $f^{-1}(x) = 5^x - 1$

B $f^{-1}(x) = \frac{5^x + 1}{2}$

C $f^{-1}(x) = \log_2(5x - 1)$

D $f^{-1}(x) = \log_5 \frac{5x + 1}{2}$

6 Two functions are shown below.

$$\begin{aligned}T(x) &= -x \\P(x) &= 10x + 2\end{aligned}$$

What is the value of $P(T(3)) - T(P(3))$?

A 8

B 4

C 0

D -4

$$\begin{aligned}& \rightarrow 32 \\& -28 - -32\end{aligned}$$

15 What is the range of the inverse of $y = \tan x$?

A $-\frac{\pi}{2} < y < \frac{\pi}{2}$

B $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

C $0 < y < \pi$

D $0 \leq y \leq \pi$

$$\left(-\frac{\pi}{2} < y < \frac{\pi}{2}\right)$$

7 A piecewise function is shown below.

$$f(x) = \begin{cases} cx + 1, & x \leq 2 \\ cx^2 - 1, & x > 2 \end{cases}$$

For what value of c does $\lim_{x \rightarrow 2} f(x)$ exist?

A -2

B -1

C 1

D 4



Cosine of a Sum or Difference

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

(Note the sign switch in either case.)

$$\begin{aligned}\sin(u+v) &= \sin u \cos v + \cos u \sin v \\ \sin(u-v) &= \sin u \cos v - \cos u \sin v\end{aligned}$$

$$\begin{aligned}\cos(u-v) &= \cos u \cos v + \sin u \sin v \\ \cos(u+v) &= \cos u \cos v - \sin u \sin v\end{aligned}$$

Write each of the following expressions as the sine or cosine of an angle.

(a) $\sin 22^\circ \cos 13^\circ + \cos 22^\circ \sin 13^\circ$ $\sin(22+13)$

(b) $\cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4}$ $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

(c) $\sin x \sin 2x - \cos x \cos 2x$ $-\cos(x+2x)$

In Exercises 1–10, use a sum or difference identity to find an exact value.

1. $\sin 15^\circ$ $(\sqrt{6} - \sqrt{2})/4$

2. $\tan 15^\circ$ $2 - \sqrt{3}$

3. $\sin 75^\circ$ $(\sqrt{6} + \sqrt{2})/4$

4. $\cos 75^\circ$ $(\sqrt{6} - \sqrt{2})/4$

5. $\cos \frac{\pi}{12}$ $(\sqrt{2} + \sqrt{6})/4$

6. $\sin \frac{7\pi}{12}$ $(\sqrt{6} + \sqrt{2})/4$



$$\frac{4\pi}{12} - \frac{3\pi}{12} =$$



Completing the Square...

$$x^2 + y^2 + 8x - 10y - 8 = 0$$

$$(x+a)^2 + (y+b)^2 =$$

$$\textcircled{1} x^2 + 8x + y^2 - 10y = 8$$

$$\textcircled{2} (x^2 + 8x + 16) + (y^2 - 10y + 25) = 8 + 16 + 25$$

$$\textcircled{3} (x+4)^2 + (y-5)^2 = 49$$

Completing the Square...

$$f(x) = 3x^2 + y^2 + 18x - 2y - 8 = 0$$

$$3x^2 + 18x \quad y^2 - 2y = 8$$

$$3(x^2 + 6x + 9) + (y^2 - 2y + 1) = 8 + 27 + 1$$

$$3(x+3)^2 + (y-1)^2 = 36$$

Completing the Square...

$$f(x) = 9x^2 - y^2 - 36x - 6y + 18 = 0$$

$$\textcircled{1} \quad 9x^2 - 36x \quad - y^2 - 6y = -18$$

$$\textcircled{2} \quad 9(x^2 - 4x + 4) - 1(y^2 + 6y + 9) = -18 \begin{matrix} +36 \\ -9 \end{matrix}$$

$$9(x-2)^2 - (y+3)^2 = 9$$

Complete the square...

1) $-x^2 + 10x + y - 21 = 0$

2) $x^2 + y^2 + 6x - 2y + 9 = 0$

3) $x^2 - y^2 - 2x - 8 = 0$

4) $9x^2 + 16y^2 + 54x - 32y - 47 = 0$

5) $-9x^2 + y^2 - 72x - 153 = 0$

6) $y^2 - 8x - 4y + 20 = 0$

$(x+3)^2 + (y-1)^2 = 1$

$(x-1)^2 - y^2 = 9$

$9(x+3)^2 + 16(y-1)^2 = 214$

$-9(x+4)^2 + y^2 = 9$

$(y-2)^2 = 8x - 16$

Complete the square...

1) $-9x^2 + 25y^2 - 100y - 125 = 0$

2) $9x^2 + 16y^2 + 54x - 32y - 47 = 0$

3) $9x^2 + 4y^2 - 54x - 8y - 59 = 0$

4) $-25x^2 + y^2 - 100x - 125 = 0$

5) $x^2 + y^2 - 9 = 0$

6) $y^2 + x + 10y + 26 = 0$

$$x^2 + y^2 + 8x - 10y - 8 = 0$$

$$4x^2 + 9y^2 - 48x + 72y + 144 = 0$$

$$5x^2 + 20y^2 + 30x + 40y - 15 = 0$$