

## Warm-up

1. Divide.  $\frac{x^4 + 10x + 2}{x + 2}$

2. Find the x and y intercepts algebraically  
 $y = 4x^2 - 5x - 6$ .

3. List all possible rational zeros.  $f(x) = 2x^4 - 3x^2 + 15$

4. Given that  $x - 4$  is a factor, find all zeros for

$$y = x^3 - x^2 - 10x - 8$$

### Homework Section 2.4

4.  $f(x) = 2x^2 - 5x + \frac{7}{2} - \frac{9/2}{2x + 1}$

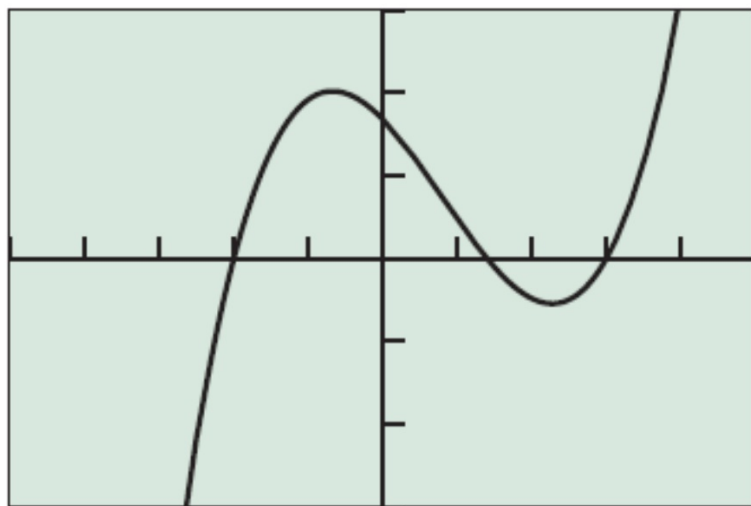
5.  $x^2 - 4x + 12 + \frac{-32x + 18}{x^2 + 2x - 1}$

11.  $\frac{5x^4 - 3x + 1}{4 - x} - 5x^3 - 20x^2 - 80x - 317 + \frac{-1269}{4 - x}$

16.  $f(x) = x^3 - 3x + 4; k = -2$  2

18.  $f(x) = x^5 - 2x^4 + 3x^2 - 20x + 3; k = -1$  23

26.  $f(x) = 5x^3 - 12x^2 - 23x + 42$   $f(x) = (x + 2)(x - 3)(5x - 7)$



$[-5, 5]$  by  $[-75, 75]$

30. Degree 4, with  $-3$ ,  $-1$ ,  $0$ , and  $\frac{5}{2}$  as zeros  $2x^4 + 3x^3 - 14x^2 - 15x$

35.  $\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}; \frac{3}{2}$

36.  $\frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 3, \pm 6}; -\frac{4}{3}$  and  $\frac{3}{2}$

50.  $f(x) = x^3 + 3x^2 - 3x - 9$  Rational zero:  $-3$ ; irrational zeros:  $\pm\sqrt{3}$

65. **Multiple Choice** Let  $f$  be a polynomial function with  $f(3) = 0$ . Which of the following statements is not true? **A**

(A)  $x + 3$  is a factor of  $f(x)$ .

(B)  $x - 3$  is a factor of  $f(x)$ .

(C)  $x = 3$  is a zero of  $f(x)$ .

(D) 3 is an  $x$ -intercept of  $f(x)$ .

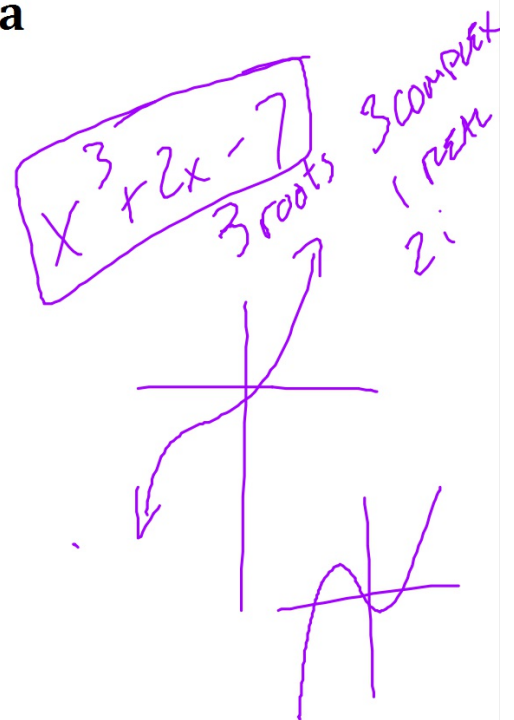
(E) The remainder when  $f(x)$  is divided by  $x - 3$  is zero.

## 2.5 Complex Zeros and the Fundamental Theorem of Algebra

Fundamental Theorem of Algebra states that:  
A polynomial function of n degree,  
has n complex roots.

This means that:

*imaginary roots  
pairs*



## Exploring Fundamental Polynomial Connections

Write the polynomial function in standard form, and identify the zeros of the function and the  $x$ -intercepts of its graph.

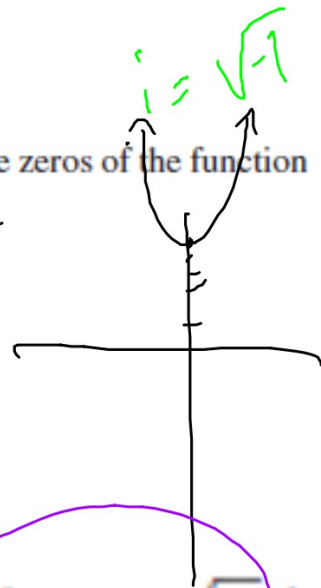
(a)  $f(x) = (x - 2i)(x + 2i)$

$$x^2 + 2ix - 2ix - 4i^2$$

$$x^2 - 4i^2$$

$$x^2 - 4(-1)$$

$$x^2 + 4$$



(b)  $f(x) = (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i)$

$$(x-5)(x^2+2) - 2i^2$$

$$x^3 - 5x^2 + 2x - 10$$

# CONJUGATES...

Suppose that  $f(x)$  is a polynomial function with *real coefficients*. If  $a$  and  $b$  are real numbers such that  $a + bi$  is a zero of  $f(x)$ , then its complex conjugate  $a - bi$  is also a zero of  $f(x)$ .

## Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include  $-3$ ,  $4$ , and  $2 - i$ .

$$\begin{aligned} &(1+3i) \\ &(1-3i) \\ &(x-(1+3i))(x-(1-3i)) \end{aligned}$$

$$(x-(2-i))(x-(2+i))$$

$$\begin{aligned} &x^2 - x(2+i) - x(2-i) + (2-i)(2+i) \\ &x^2 - 2x - xi - 2x + xi + 4 - i^2 \\ &x^2 - 4x + 5 \end{aligned}$$

$$(x+3)(x-4)$$

**Ex.** write a polynomial function of minimum degree in standard form with real coefficients whose zeros and their multiplicities include those listed.

13. 1 (multiplicity 2),  $-2$  (multiplicity 3)

In Exercises 21–26, state how many <sup>4</sup> complex and <sup>2</sup> real zeros the function has.

25.  $f(x) = x^4 - 5x^3 + x^2 - 3x + 6$



### Finding Complex Zeros

The complex number  $z = 1 - 2i$  is a zero of  $f(x) = 4x^4 + 17x^2 + 14x + 65$ . Find the remaining zeros of  $f(x)$ , and write it in its linear factorization.

**Hi 4 0 17 14 65. , unit Hista  
1+z : y%f%i\*1 -**

| |  
| |  
| |  
| |

**You try:**

$$\mathbf{x^5 - 6x^4 + 11x^3 - x^2 - 14x + 5}$$

**Is  $(2 - i)$  a solution?**

### **Polynomial Function of Odd Degree**

Every polynomial function of odd degree with real coefficients has at least one real zero.

## Wrap-up

What is the polynomial of least degree that has the roots  $-2, 3i$  ?



In Exercises 33–36, using the given zero, find all of the zeros and write a linear factorization of  $f(x)$ .

**33.**  $1 + i$  is a zero of  $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$ .

**34.**  $4i$  is a zero of  $f(x) = x^4 + 13x^2 - 48$ .

**35.**  $3 - 2i$  is a zero of  $f(x) = x^4 - 6x^3 + 11x^2 + 12x - 26$ .

**36.**  $1 + 3i$  is a zero of  $f(x) = x^4 - 2x^3 + 5x^2 + 10x - 50$ .

# **Relay Race**

**Rules...**

**Is the following binomial a factor?**

**1)  $x^4 + 3x^3 + 3x^2 + 23x - 20$  ;  $(x + 4)$**

**2)  $8x^3 - 22x^2 - 9x + 9$  ;  $(x - 3)$**

**3)  $3x^3 - 75$  ;  $(x - 3)$**

**4)  $x^3 - 30x - 36$  ;  $(x - 6)$**

**5)  $7x^3 + 4x^2 + 3$  ;  $(x - 1)$**

**6)  $9x^3 + 63x^2 - 78x - 49$  ;  $(x + 8)$**

Use  $f(x) = 2x^2 - x$  and  $g(x) = 2x + 2$

1)  $(f - g)(-2)$

2)  $(g + f)(1)$

3)  $(f + f)(-3)$

4)  $(g - f)(2)$

5)  $(g + g)(4)$

6)  $(f - g)(-1)$

Use  $f(x) = 4x - 3$  ;  $g(x) = 2x + 2$  ;  $h(x) = x - 5$

1)  $(f(g(x)))$

2)  $(h(f(x)))$

3)  $(f \circ h)(x)$

4)  $(g \circ f)(x)$

5)  $(h(g(x)))$

6)  $(f \circ h(x))$