Warm up

1. If \( f(t) = 3t^2 - 2 \), find \( f(k - 2) \).

   (A) \( 3k^2 - 12k + 12 \)  
   (B) \( 3k^2 - 8 \)  
   (C) \( 3k^2 - 4k + 2 \)  
   (D) \( 3k^2 - 4 \)  
   (E) None of these

An equation of a graph obtained from vertically shrinking the graph of \( y = \sqrt{x} \) then shifting the graph up twenty units is:

2. \( y = \frac{5}{3} \sqrt{x} + 20 \)  
   \( y = \sqrt{\frac{7}{2} x + 20} \)  
   \( y = \frac{3}{4} \sqrt{x} + 20 \)  
   (D) \( y = 2 \sqrt{x} + 20 \)  
   (E) None of these

3. Find \( Q^{-1}(t) \) if \( Q(t) = \frac{C}{4t - 1} \). (\( C \) is a nonzero real number)

   (A) \( Q^{-1}(t) = \frac{C}{4} t + C \)  
   (B) \( Q^{-1}(t) = \frac{4t - 1}{C} \)  
   (C) \( Q^{-1}(t) = \frac{C + t}{4t} \), \( t \neq 0 \)  
   (D) \( Q^{-1}(t) = \frac{C - 4t}{t}, t \neq 0 \)  
   (E) None of these

Homework
Functions
Logistic Growth Functions

In this lesson you will study a family of functions of the form

\[ y = \frac{c}{1 + ae^{-rx}} \]

where \( a, c, \) and \( r \) are all positive constants. Functions of this form are called logistic growth functions.

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**GRAPHS OF LOGISTIC GROWTH FUNCTIONS**

The graph of \( y = \frac{c}{1 + ae^{-rx}} \) has the following characteristics:

- The horizontal lines \( y = 0 \) and \( y = c \) are asymptotes.
- The \( y \)-intercept is \( \frac{c}{1 + a} \).
- The domain is all real numbers, and the range is \( 0 < y < c \).
- The graph is increasing from left to right. To the left of its point of maximum growth, \( \left( \frac{\ln a}{r}, \frac{c}{2} \right) \), the rate of increase is increasing. To the right of its point of maximum growth, the rate of increase is decreasing.
The maximum height, in inches, a ball reaches after its first four bounces is shown in the table below.

<table>
<thead>
<tr>
<th>Bounce Number</th>
<th>Height (in inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.0</td>
</tr>
<tr>
<td>2</td>
<td>31.5</td>
</tr>
<tr>
<td>3</td>
<td>23.6</td>
</tr>
<tr>
<td>4</td>
<td>17.7</td>
</tr>
</tbody>
</table>

Which type of function best models the data and why?

A. an exponential function, because the height of the ball is decreasing by 25% with each bounce

B. an exponential function, because the height of the ball is decreasing by 75% with each bounce

C. a logistic function, because the height of the ball is decreasing by 25% with each bounce

D. a logistic function, because the height of the ball is decreasing by 75% with each bounce

Transformations of the graphs of functions

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) + c$</td>
<td>shift $f(x)$ up $c$ units</td>
</tr>
<tr>
<td>$f(x) - c$</td>
<td>shift $f(x)$ down $c$ units</td>
</tr>
<tr>
<td>$f(x + c)$</td>
<td>shift $f(x)$ left $c$ units</td>
</tr>
<tr>
<td>$f(x - c)$</td>
<td>shift $f(x)$ right $c$ units</td>
</tr>
<tr>
<td>$f(-x)$</td>
<td>reflect $f(x)$ about the y-axis</td>
</tr>
<tr>
<td>$-f(x)$</td>
<td>reflect $f(x)$ about the x-axis</td>
</tr>
</tbody>
</table>
| $cf(x)$       | When $0 < c < 1$ – vertical shrinking of $f(x)$
|               | When $c > 1$ – vertical stretching of $f(x)$
|               | Multiply the y values by $c$ |
| $f(cx)$       | When $0 < c < 1$ – horizontal stretching of $f(x)$
|               | When $c > 1$ – horizontal shrinking of $f(x)$
|               | Divide the x values by $c$ |
1. What transformations have occurred to create the function \( f(x) = 3x^3 - 4 \) from the function \( g(x) = x^3 \)?
   
   A. The graph of the function has been stretched horizontally and shifted up four units.
   
   B. The graph of the function has been stretched vertically and shifted up four units.
   
   C. The graph of the function has been stretched horizontally and shifted down four units.
   
   D. The graph of the function has been stretched vertically and shifted down four units.

   - [D]  

3. What is the range of the function \( f(x) = -5 - 2(x + 3)^2 \)?
   
   A. \([-5, \infty)\)
   
   B. \((-\infty, 5]\)
   
   C. \((-\infty, -5]\)
   
   D. \((-\infty, \infty)\)

   - [C]  

14. Match the transformations that would create the graph of \( g(x) \) from the graph of \( f(x) \).

   - [B] \( g(x) = 3f(x) \)  
   - [C] \( g(x) = f(3x) \)  
   - [A] \( g(x) = f\left(\frac{1}{3}x\right) \)  
   - [D] \( g(x) = \frac{1}{3}f(x) \)

   A. Stretch the graph of \( f(x) \) horizontally
   
   B. Stretch the graph of \( f(x) \) vertically
   
   C. Shrink the graph of \( f(x) \) horizontally
   
   D. Shrink the graph of \( f(x) \) vertically

For items 5 and 6, solve the equations. Check for any extraneous solutions.

5. \( \frac{3x}{x-1} = \frac{12}{x^2 - 1} + 2 \)
   
   \[ x = -5, x = 2 \]

6. \( \frac{x}{x-2} + \frac{3x}{x-4} = \frac{32 - 2x}{x^2-6x+8} \)
   
   \[ x = -2 \] (\(-4\) is extraneous)
1. Which of the following is equivalent to $\log_5 \left( \frac{5}{x^3} \right)$?

A. $5 - 3\log_5 x$

B. $1 - 3\log_5 x$

C. $-3\log_5 x$

D. $2\log_5 x$

#2 & 3. Solve log equations

$$\log_7 (3x - 1) = 2$$

$$\log_5 x + \log_5 (x - 4) = 1$$

4. A colony of insects has an initial population of 600. The number of insects triples every 4 weeks.

a. Write a function for the number of insects $N(t)$ after $t$ weeks.

b. What will the number of insects (nearest whole number) be after 7 weeks?

c. About how many weeks (correct to three decimal places) will the number of insects be 12,000?