Student Name: ____________________________

Spring 2013
North Carolina
Measures of Student Learning:
NC’s Common Exams
Precalculus

Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314

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1 What are the approximate rectangular coordinates for the point with polar coordinates (5, 30°)?

A  (2.5, 2.89)
B  (2.5, 4.33)
C  (2.89, 4.33)
D  (4.33, 2.5)

2 A sequence is shown below.

\[6, 12, 20, 30, 42, 56, \ldots\]

Which is the recursive formula for this sequence?

A  \(t_n = n + 2(t_{n-1} + 1)\)
B  \(t_n = (t_{n-1} + 1)(n - 2)\)
C  \(t_n = 2(t_{n-1} + 2) - (n + 2)\)
D  \(t_n = t_{n-1} + 2(n + 1)\)
3 A quadratic function, \( f \), has zeros \( P \) and \( Q \), such that \( P + Q = 5 \) and \( \frac{1}{P} + \frac{1}{Q} = 8 \).
Which choice describes \( f \)?

A \( f(x) = 8x^2 - 40x + 5 \)
B \( f(x) = 8x^2 - 40x - 5 \)
C \( f(x) = 2x^2 - 10x + 5 \)
D \( f(x) = 2x^2 - 10x - 5 \)

4 Lucy invested $6,000 into an account that earns 6% interest compounded continuously. *Approximately* how long will it take for Lucy’s investment to be valued at $25,000?

A 52.7 years
B 46.9 years
C 24.5 years
D 23.8 years

5 A lamppost is located 418 feet from a building. The angle of elevation from the base of the lamppost to the top of the building is 32.3°. *Approximately* how tall is the building?

A 223 feet
B 264 feet
C 510 feet
D 661 feet
6. Two functions are shown below.

\[ T(x) = -x \]
\[ P(x) = 10x + 2 \]

What is the value of \( P(T(3)) - T(P(3)) \)?

A. 8  
B. 4  
C. 0  
D. -4

7. A piecewise function is shown below.

\[ f(x) = \begin{cases} 
  cx + 1, & x \leq 2 \\
  cx^2 - 1, & x > 2 
\end{cases} \]

For what value of \( c \) does \( \lim_{{x \to 2}} f(x) \) exist?

A. -2  
B. -1  
C. 1  
D. 4
8 What are the polar coordinates of (4, 9)?

A \((\sqrt{97}, 66^\circ)\)

B \((\sqrt{97}, 114^\circ)\)

C \((\sqrt{13}, 66^\circ)\)

D \((\sqrt{13}, 114^\circ)\)

9 A sequence is shown below.

\[1, 3, 3^2, 3^3, \ldots\]

How many terms of the sequence must be added together for the sum to equal 3,280?

A 6

B 7

C 8

D 9
10 The first term of an infinite geometric sequence is 2. The sum of the sequence is 6. What is the common ratio of the sequence?

A \( \frac{1}{3} \)

B \( \frac{2}{3} \)

C \( \frac{3}{3} \)

D \( \frac{4}{3} \)

11 Which is true of the series shown below?
\[
\pi + \frac{3\pi}{4} + \frac{9\pi}{16} + \frac{27\pi}{64} + \ldots
\]

A The series diverges.

B The series converges to \( \frac{3\pi}{2} \).

C The series converges to \( \frac{4\pi}{3} \).

D The series converges to \( 4\pi \).
12 Karen recursively generated a sequence of five positive integers by starting with a positive integer, \( a_1 \), and then applying the recursive formula \( a_n = a_{n-1} + 3n - 1 \) to generate \( a_n \) for \( n = 2, 3, 4, \) and \( 5 \).

If the value of \( a_5 \) was 407, what was the value of Karen’s starting term, \( a_1 \)?

A 366  
B 367  
C 368  
D 369

13 What is the distance between \( y \)-intercepts of the graph of \( x + 8 = 2(y + 3)^2 \)?

A 4  
B 6  
C 11  
D 15

14 Which is a solution set to \( x + \frac{3x}{x - 1} = \frac{x + 2}{x - 1} \)?

A \{-1\}  
B \{-2\}  
C \{-2, 1\}  
D \{2, -1\}
15 What is the range of the inverse of $y = \tan x$?

A $\frac{-\pi}{2} < y < \frac{\pi}{2}$

B $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

C $0 < y < \pi$

D $0 \leq y \leq \pi$

16 James is standing 10 meters away from Samantha.

- A bird is located in the sky at a point between where James and Samantha are standing.
- James is looking up at the bird at an angle of elevation of 74°.
- Samantha is looking up at the bird at an angle of elevation of 47°.

**Approximately** how far is the bird from Samantha?

A 7.6 meters

B 8.5 meters

C 11.2 meters

D 13.1 meters
17. What is the inverse function of \( f(x) = \log_5 (2x - 1) \)?
   A. \( f^{-1}(x) = 5^x - 1 \)
   B. \( f^{-1}(x) = \frac{5^x + 1}{2} \)
   C. \( f^{-1}(x) = \log_2(5x - 1) \)
   D. \( f^{-1}(x) = \log_5 \left( \frac{5x + 1}{2} \right) \)

18. What is the value of the limit shown below?
   \[ \lim_{n \to \infty} \left( \frac{3^n - 1}{3^n} \right) \]
   A. \( \frac{1}{3} \)
   B. \( \frac{2}{3} \)
   C. 1
   D. \( +\infty \)

19. What type of conic section is represented by \( r = \frac{8}{16 + 125 \sin \theta} \)?
   A. circle
   B. ellipse
   C. hyperbola
   D. parabola
20 James had a rectangular piece of cardboard that was four times as long as it was wide. He wanted to use the cardboard to make a box with no lid. To do this, he first cut a 3-by-3-inch square out of each of the four corners of the piece of cardboard, as shown in the picture below.

Then James folded the cardboard along the four dotted lines shown in the picture. This created an open box with a volume of 336 cubic inches.

What was the width of the sheet of cardboard that James started with?

A 10.5 inches
B 9.5 inches
C 8.5 inches
D 7.5 inches

21 Which expression is equivalent to \((\sec \theta) \left( \frac{\sin \theta}{\tan \theta} \right)\)?

A \(\cos^2 \theta - \sin^2 \theta\)
B \(\sin^2 \theta - \cos^2 \theta\)
C \(\cot^2 \theta - \csc^2 \theta\)
D \(\csc^2 \theta - \cot^2 \theta\)
22. Suppose that for each foot of land along the street, the annual tax is $25 per foot. The diagram below shows a plot of land.

![Diagram of a plot of land with dimensions 80°, 28 ft, and 46 ft]

**About** how much is the annual tax for the plot?

A. $1,238  
B. $1,293  
C. $1,321  
D. $1,411

23. The function $C(x) = \frac{2.50x + 1.00}{x}$ models the cost per item for a company to produce $x$ items after the first item is made. What is the inverse function of $C(x)$?

A. $C^{-1}(x) = \frac{1.00}{x - 2.50}$  
B. $C^{-1}(x) = \frac{x - 2.50}{1.00}$  
C. $C^{-1}(x) = \frac{x - 1.00}{2.50}$  
D. $C^{-1}(x) = \frac{2.50}{x - 1.00}$
24 A computer rental company charges $50 to rent a computer for one week. The table below shows the daily late fees the company charges if a computer is returned late.

<table>
<thead>
<tr>
<th>Days Late</th>
<th>Daily Late Fee</th>
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<tbody>
<tr>
<td>days 1 through 10</td>
<td>$5</td>
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<tr>
<td>days 11 through 20</td>
<td>$8</td>
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<tr>
<td>days 21 through 30</td>
<td>$10</td>
</tr>
</tbody>
</table>

What would be the total cost of renting a computer for one week and returning it 15 days late?

A $120  
B $125  
C $140  
D $170

25 From a point 100 feet from the base of a building, Angie looks up at a 40° angle to the top of a building. She walks 20 feet closer to the building. At approximately what angle must Angie now look up to see the top of the building?

A 32°  
B 46°  
C 60°  
D 77°

This is the end of the multiple-choice portion of the test.
The questions you read next will require you to answer in writing.

1. Write your answers on separate paper.

2. Be sure to write your name on each page.

1. The equation \( y = \frac{1}{18}x^2 \) represents the mirror inside a parabolic lamp.
   - What is the focal width of the mirror?
   - Use the equation to explain your answer.

2. The function \( P(t) = 1,440e^{-0.0259t} \) models the number of cars a dealership sold \( t \) years after the first year it was open.
   - By what percent is the number of cars being sold decreasing each year?
   - How many cars did the dealership sell the year it opened?

3. Two parametric equations are shown below.
   \[
   x = \frac{3t^2}{2} \\
   y = 4t - 1
   \]
   - Convert the parametric equations into rectangular form.
   - Determine what type of equation the rectangular form describes.
This is the end of the Precalculus test.

1. Look back over your answers.

2. Put all of your papers inside your test book and close the test book.

3. Place your calculator on top of the test book.

4. Stay quietly in your seat until your teacher tells you that testing is finished.
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<thead>
<tr>
<th>Item number</th>
<th>Type</th>
<th>Key</th>
<th>Competency Goal(s)</th>
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<td>1</td>
<td>MC</td>
<td>D</td>
<td>2 — Relations and Functions</td>
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## Item Types:
MC = multiple choice  
CR = constructed response

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